Homework 3

(Total 170 pts)

Due at 5:00 pm on October 11, 2024 (Friday)

Note: Make sure you convert the scanned images into black-and-white images, before converting them to a single PDF file for Canvas upload. Also make sure the texts are clearly visible. Failure to do so will result in points being deducted.

1. (40 pts) A random variable X has a cumulative distribution function (CDF) of the form:

$$F_X(x) = \begin{cases} 0 & -\infty < x \le -2 \\ \frac{1}{2}\left[1 + \sin\left(\frac{\pi}{4}x\right)\right] & -2 < x \le 2 \\ 1 & 2 < x \le \infty \end{cases}.$$

   (a) Determine the probability density function (PDF) of X, $f_X(x)$.

   (b) Show analytically that $\int_{-\infty}^{+\infty} f_X(x)dx = 1$.

   (c) In Matlab, plot the waveform of $f_X(x)$, where $-2 < x \le 2$, with a step size of 0.01. Then use the function *trapz* to find the numerical value of the integration $\int_{-2}^{2} f_X(x)\,dx$.

   (d) Attach a screenshot showing the Matlab script, the plot, and the numerical result of $\int_{-2}^{2} f_X(x)\,dx$ after running the script.

2. (30 pts)
   Exercise 3-2.1 (pp 129). Sub-problem a) only. Show your detailed derivations to get full credit.

3. (20 pts)
   Problem 3-4.1 (pp 154). Sub-problems a) and b) only.

4. (40 pts)
   Two random variables X and Y have a joint probability density function given by

$$f_{XY}(x,y) = \begin{cases} x + y, & \text{if } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

   (a) Determine the numerical value of $E[XY]$.

   (b) Determine the numerical value of the correlation coefficient ρ_{XY}.

   Show your detailed derivations to get full credit. Round your answer to the fourth place to the right of the decimal point.

5. (40 pts) Matlab programming to calculate the correlation coefficient between two color components.

   (a) Use the *imread* function to read in the built-in image 'football.jpg' on Matlab and convert the data type to 'double' as follows:

   ```
   >> I = imread('football.jpg');
   >> I = double (I);
   ```

   (b) Write a Matlab script to calculate the following quantities:
   Expected value of the red component: $E[R]$;
   Standard deviation of the red component: σ_R;

Expected value of the blue component: $E[B]$;
Standard deviation of the blue component: σ_B;
Correlation of the red and blue components: $E[RB]$;
Correlation coefficient of the red and blue components: ρ_{RB}.

Fill in the table below with the corresponding values.
Note: failure to put the results in the table will result in points being deducted.

	$E[R]$	σ_R	$E[B]$	σ_B	$E[RB]$	$\rho_{RB} = \frac{E[RB] - E[R]E[B]}{\sigma_R \sigma_B}$
Values						

(c) Now use the image you obtained for Question 5 in HW 1 to re-calculate the above quantities and fill in another table with the corresponding values. In addition, display and attach the red and blue components of your image.
(d) Attach your Matlab script and a screenshot showing the result of running the script.