Homework 4

(Total 110 pts)

Due 5:00 pm on November 8, 2024 (Friday)

Note: Make sure you convert the scanned images into black-and-white images, before converting them to a PDF file for Canvas upload. Also make sure the texts are clearly visible. Failure to do so will result in points being deducted.

1. (20 pts) A random variable X has a PDF $f_X(x) = 2e^{-2x}u(x)$. Using the characteristic function to find $E[X^2]$.

2. (20 pts) Let X and Y be two statistically independent random variables having probability density functions: $f_X(x) = 2e^{-2x}u(x)$, and $f_Y(y) = 2e^{-2y}u(y)$. Let the random variable $Z = X + Y$. Determine $E[Z^2]$ using characteristic functions.

3. (20 pts) Exercise 4-2.2 (pp 166). Sub-problem a) only. You need to show the derivations to get full credit.

4. (30 pts) Let X and Y be two statistically independent random variables having probability density functions: $f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$, and $f_Y(y) = \begin{cases} 1, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$. Let the random variable $Z = X + Y$.
   (a) Determine analytically the probability density function (PDF) of Z, $f_Z(z)$.
   (b) In Matlab, plot the waveform of $f_Z(z)$. Attach the Matlab script and the plot.

5. (20 pts) In Matlab, use the **rand** function to generate a vector (*X)* of one million random numbers uniformly distributed between 0 and 1. Then use the **rand** function again to generate another vector (*Y*) of one million random numbers also uniformly distributed between 0 and 1. Let $Z = X + Y$.
   (a) Plot the normalized histogram of Z (using the option: 'Normalization', 'pdf'). Attach the Matlab script and the histogram.
   (b) Compare the normalized histogram of Z with the waveform of $f_Z(z)$ obtained in Problem 4. Comment on your result.