Lecture 19

For example : $X: N(0, \sigma_X)$, $Y: N(0, \sigma_Y)$, $Z = X+Y$; X and Y are independent.

Question : $f_Z(z) = ?$

Answer :

$$X: N(0, \sigma_X) \longrightarrow \phi_X(u) = e^{-\frac{u^2\sigma_X^2}{2}}$$

$$Y: N(0, \sigma_Y) \longrightarrow \phi_Y(u) = e^{-\frac{u^2\sigma_Y^2}{2}}$$

$$Z: \quad \phi_Z(u) = \phi_X(u)\cdot\phi_Y(u) = e^{-\frac{u^2\sigma_X^2}{2}}\cdot e^{-\frac{u^2\sigma_Y^2}{2}}$$

$$= e^{-u^2(\sigma_X^2+\sigma_Y^2)/2}$$

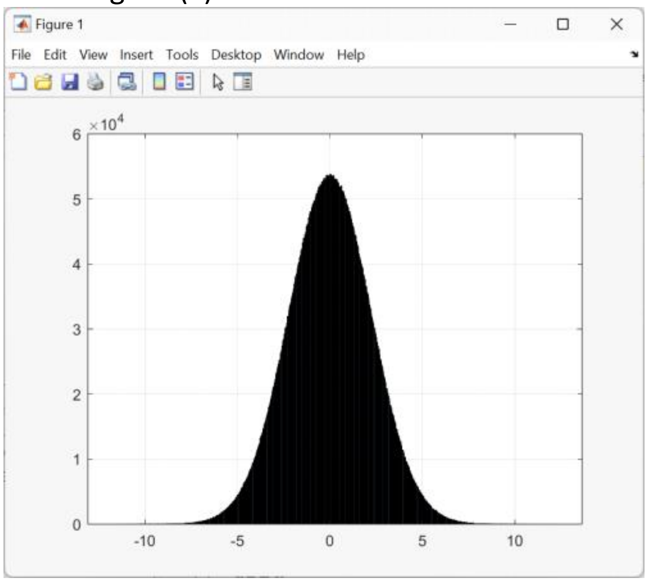
$$\Downarrow$$

$$Z: N(0, \sqrt{\sigma_X^2+\sigma_Y^2})$$

```
>> X = randn(1, 10000000);
>> Y = 2*randn(1, 10000000);
>> Z = X+Y;
>> var(Z)
ans =
   4.9996

>> var(X) + var(Y)
ans =
   4.9999
```

```
>> std(Z)
ans =
   2.2360

>> sqrt(var(X) + var(Y))
ans =
   2.2360
```

```
>> histogram(Z)
```



Normality Check:

```
>> S = find(abs(Z - mean(Z)) <= std(Z));
>> length(S)/length(Z)
ans =
   0.6826

>> S = find(abs(Z - mean(Z)) <= 2*std(Z));
>> length(S)/length(Z)
ans =
   0.9545

>> S = find(abs(Z - mean(Z)) <= 3*std(Z));
>> length(S)/length(Z)
ans =
   0.9973
```

In general, if $Y = X_1 + X_2 + \cdots + X_n$

$$\phi_Y(u) = \phi_{X_1}(u) \cdot \phi_{X_2}(u) \cdots \phi_{X_n}(u)$$

Chapter 4: Elements of Statistics

Sampling Theory: Selecting samples from a collection of data that is too large to be examined individually.

Question: determine how many samples are required for a given degree of confidence in the result.

Population (N): Collection of data being studied.
Sample Size (n), where sample is part of the population selected at random

- Sample Mean

$X_1, X_2, \ldots, X_n$ random variables (all having the same $\bar{X}$)

True Mean

Define Sample Mean as: $$\hat{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

... the Sample Mean as: $\hat{X} = \frac{1}{n} \sum_{i=1} X_i$ True Mean

also a RV

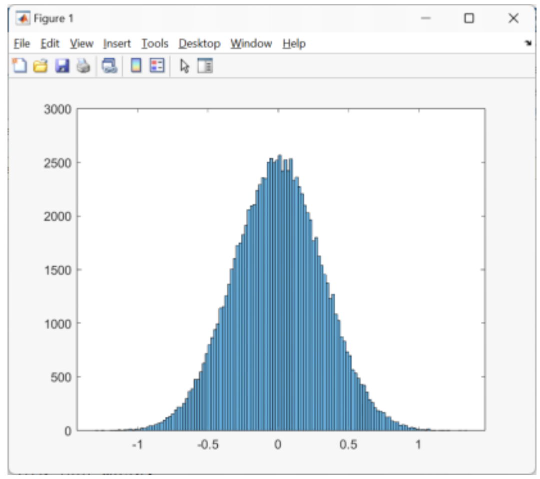
$$E[\hat{X}] = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n}\cdot \sum_{i=1}^{n} \overline{X} = \frac{1}{n}\cdot n \cdot \overline{X} = \overline{X}$$

True Mean

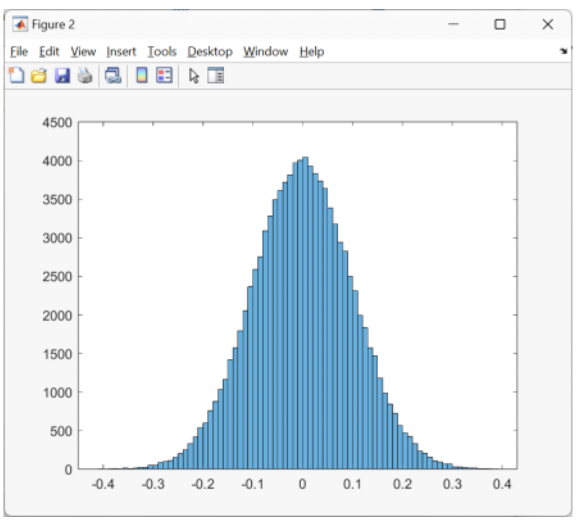
Mean of the Sample Mean = True Mean

http://www.ece.uah.edu/~dwpan/course/ee385/code/

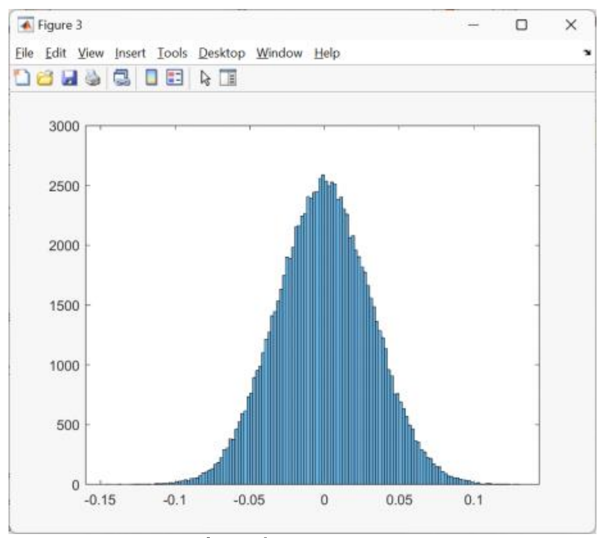
```
% sample_mean.m

sample_size = 10
run = 100000
Xavg = zeros(1, run);
for i = 1: run
   X = randn (1, sample_size);
   Xavg(i) = mean (X);
end
disp('E[X_bar] =');
mean(Xavg)
disp('Var[X_bar] =');
var(Xavg)
```



sample_size = 10, Var[X_bar] = 0.0995
Mean[X_bar] = -0.0015



sample_size = 100, Var[X_bar] = 0.0100



sample_size = 1000, Var[X_bar] = 0.0010

- Variance of the Sample Mean
  (1) Sample with Replacement: N >> n
  (2) Sample without Replacement: N is small