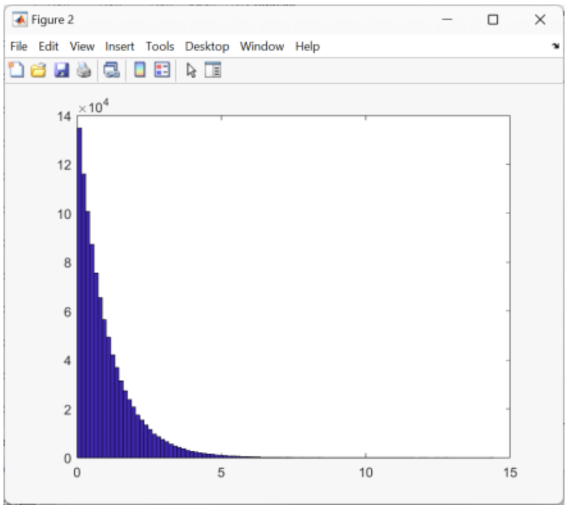
Lecture 21

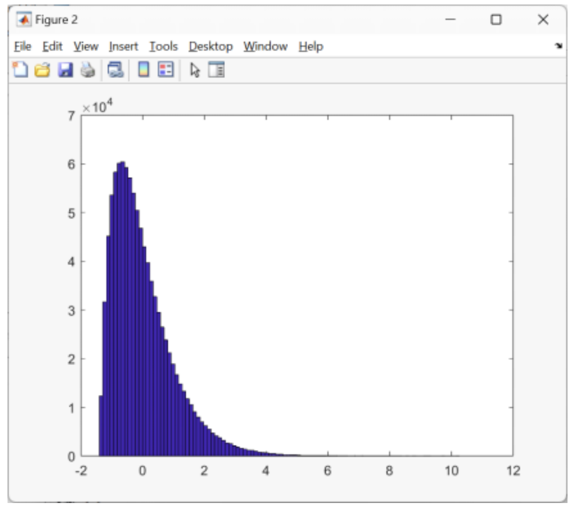
Central Limit Theorem (cont'd)

http://www.ece.uah.edu/~dwpan/course/ee385/code/clt_hist.m

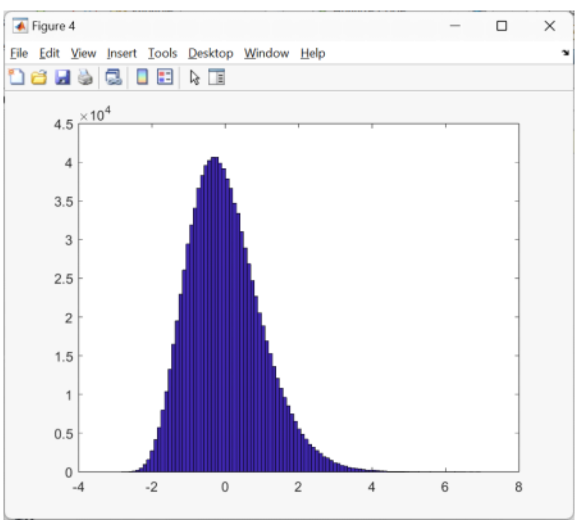
```
function [m, v] = clt_hist(n)
```



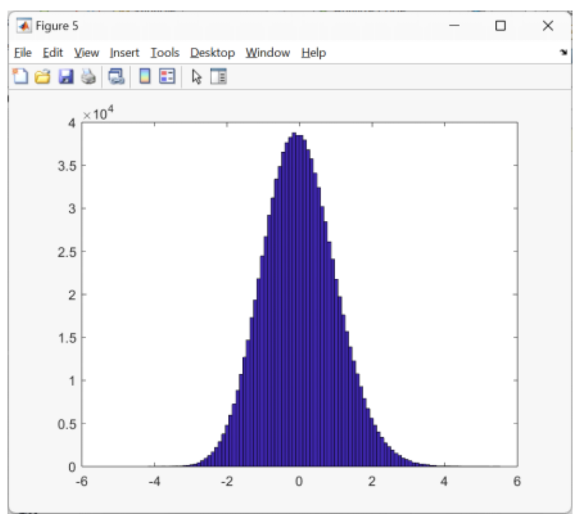
$n = 1$



$n = 2$



$n = 10$



$n = 100$

m =
  -3.9108e-17
v =
   0.9990

'Normality Check'

normal distribution: approximately 68%, 95%, and 99.7% of the values lie within one, two, and three standard deviations.

Chapter 5 Random Processes (RP's)

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

deterministic function of time t : $x(t) = \sin(\omega t)$

ω: constant

$\downarrow$

Random Process : $X(t)$

- Example of Random Process : $X(t) = A\cos(\omega t)$

A is a random variable, uniform distributed between $(0, 1)$, ω is a constant (e.g., $\omega = 1$)

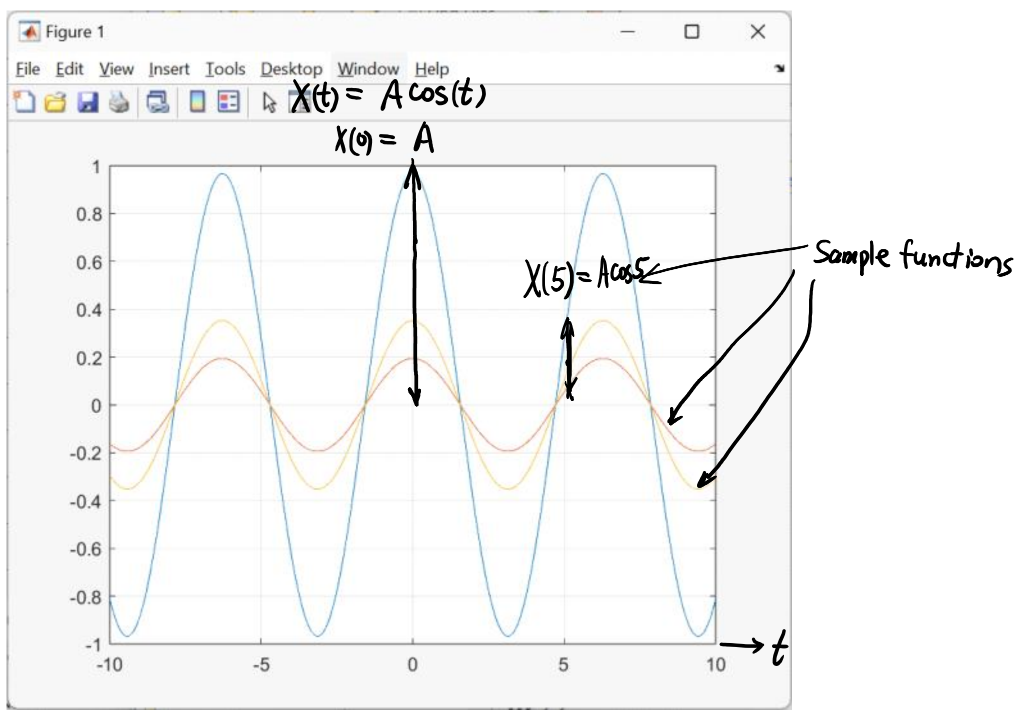
$X(t) = A\cos t$

```
>> t = -10: 0.01: 10;
>> A = rand

A =
   0.9673

>> X = A*cos(t);
>> plot(t, X)
>> grid
>> A = rand
A =
   0.1941
>> X = A*cos(t);
>> hold on;
>> plot(t, X)
>> A = rand

A =
   0.3522
>> X = A*cos(t);
>> hold on;
>> plot(t, X)
```



- Ensemble
  The entire collection of time function {x(t)}, where x(t) is a sample function of the ensemble
- X{ti} is a random variable

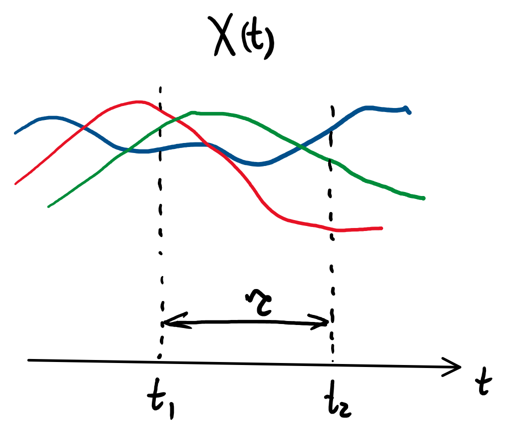
- Stationary RP's

(1) All marginal and joint PDF's do not depend on the choice of the time origin.
(2) All of the mean values and moments are constants.

Relax the above conditions slightly to define W.S.S RP's
"Wide-Sense Stationary Random Processes"
(1) Mean value of X(ti) is a constant, independent of the choice of ti;
(2) Correlation of the two random variables X(ti) and X(ti + Δt) depends only on Δt.

$$E\left[X(t_i)\,X(t_i+\Delta t)\right] = E\left[X(t_1)\cdot X(t_1+\Delta)\right] = E\left[X(t_2)\,X(t_2+\Delta)\right]$$

Chapter 6: Correlation Functions



Auto**c**orrelation **F**unction (ACF)

$$R_X(t_1,t_2) = E\left[X(t_1)\cdot X(t_2)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, f_X(x_1,x_2)\, dx_1\, dx_2$$

For W.S.S RP's :

$$R_X(t_1,t_2) = R_X(t_1+\Delta,\, t_2+\Delta) = R_X(t_2-t_1) = R_X(\tau)$$

Also,

$$E\left[X(t_1)\right] = E\left[X(t_2)\right] = E\left[X(t)\right] : \text{Constant}$$

$$E\left[X^2(t_i)\right] = E\left[X(t_i)\,X(t_i)\right] = R_X(t_i-t_i) = R_X(0)$$

Properties of Autocorrelation Functions (W.S.S RP's)

1. $R_X(t_1, t_2) = R_X(t_2 - t_1)$

$$R_X(0) = E[X^2(t)] = E[X^2]$$

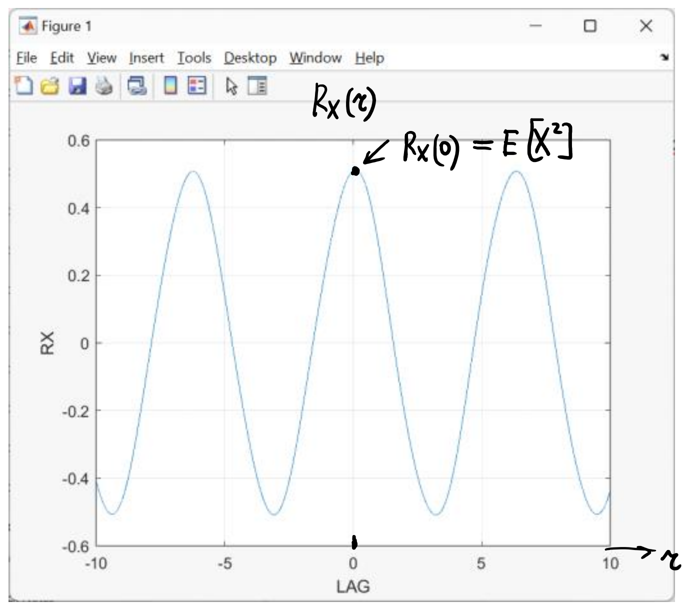
http://www.ece.uah.edu/~dwpan/course/ee385/code/corrx_example4.m

```
% corrx_example4.m:
% Example of autocorrelation estimation.
% Still use time averaging, but do not use corb( ) this time.
% Get rid of the finite-duration effect of convolution.
% x(t) = cos(t + theta), where theta is uniformly distributed.

sample_period = 0.001;
tmin = -10;         % Time shift range
tmax = 10;
theta = random('Uniform', 0, 2*pi);
tau = -tmax: sample_period: tmax;
M = length(tau);
for i = 1: M
    if tau(i) >= 0
        t = tmin: sample_period: (tmax + tau(i));
        xt = cos(t + theta);
        xtau = cos(t + tau(i) + theta);
        R(i) = mean(xt .* xtau);
    else
        t = (tmin +tau(i)): sample_period: tmax;
        xt = cos(t + theta);
        xtau = cos(t + tau(i) + theta);
        R(i) = mean(xt .* xtau);
    end
end
plot(tau,R);
xlabel('LAG'); ylabel('RX'); grid;
```

$X(t) = \cos(t + \theta)$

θ: uniformly distributed $(0, 2\pi)$



2. $R_X(\tau) = R_X(-\tau) \Rightarrow$ ACF is an even function

Why?

$$R_X(\tau) = E\left[X(t) \cdot X(t+\tau)\right]$$