Homework 4

(Total 200 pts)

Due 5:00 pm, December 6, 2024 (Friday)

Canvas submission as a single PDF file ('hw4_lastname.pdf'), along with the Matlab scripts (Q1.m, Q2.m, Q4.m, and Q5.m)

- (50 pts) Revisit Question 2 in HW 3: Mini-project on blending images to create a composite image by superimposing a portrait on a blurred background. The smoothing filter will be implemented this time in the frequency domain using the Gaussian low-pass filter (LPF). Use zero padding before DFT domain operations. Compare the various blurring effects by using three distinct standard deviation σ values (1%, 5% and 10% of the padded image size) for the Gaussian filter. Note: if the background image is a color image, then apply the same filter on each of the three color channels (R, G, and B) separately, in order to generate the low-pass filtered image in color. Also, you might want to convert the blurred image to uint8 type before overlaying the portrait on it.
 - (a) Display the DFT spectrum (centered, and using log function to enhance the contrast) of the background image). If the image is a color image, display the DFT magnitude spectrum for each of the three color components (channels).
 - (b) Display the mesh plots for the Gaussian LPF transfer functions H(u, v) (centered) for different σ values.
 - (c) Display the composite images generated for different σ values.
 - (d) Upload the Matlab scripts you used (named "Q1.m") to Canvas.
- 2. (40 pts) In Matlab, load the built-in image "coins.png" to the variable f.
 - (a) Display the image *f*.
 - (b) Display the centered DFT spectrum |F(u, v)| (use log function to enhance the contrast).
 - (c) Given the following 3×3 filter *h* in the spatial domain, display the filter in the frequency domain using **freqz2** as a mesh plot (by turning the axis off).

$$h = \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{2}{3} & -\frac{10}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}.$$

- (d) Display |H| as an image, where *H* is the frequency-domain representation of the filter *h* after zero-padding according to the size of image *f*. Center the |H| image.
- (e) Display |H| as an image, but this time the origin is at top left of the frequency rectangle.
- (f) Filter the image f in the spatial domain by h. Zero-pad the image with 0s before filtering. Display the filtering result in spatial domain g_s as an image.
- (g) Filter the image f in the frequency domain by H. Display the filtering result in spatial domain g_f as an image.
- (h) Calculate the largest and smallest values of the absolute difference image $|g_f g_s|$.
- (i) Attach the Matlab script and the images displayed.
- (j) Attach a screenshot of running your Matlab script to obtain the results above.

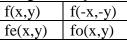
- (k) Upload the script M file (named "Q2.m") to Canvas.
- 3. (20 pts) Energy Conservation for Unitary Transforms. Suppose an N × N matrix A has the property that A^TA = AA^T = I, where I is the identity matrix. If X₁ and X₂ are two N-dimensional column vectors and Θ₁ = AX₁
 Θ₂ = AX₂
 Show analytically that |X₁ - X₂|² = |Θ₁ - Θ₂|², where |Y| denotes the L-2 norm (also called Euclidean length) of a N-dimensional vector Y.
- 4. (50 pts) Lossy Signal Compression using DCT and Distortions.
 - (a) In Matlab, run "A = dctmtx (4)" to generate the 4 × 4 Discrete Cosine Transform (DCT) matrix. What are the entries of the matrix A?
 - (b) Suppose $X_1 = [1 2 3 4]^T$ and $X_2 = [1 1 1 1]^T$. Fill in the following table with the values of the quantities:

| $ X_1 ^2$ | $ AX_1 ^2$ | $ X_2 ^2$ | $ AX_2 ^2$ | $ X_1 - X_2 ^2$ | $ AX_1 - AX_2 ^2$ |
|-----------|------------|-----------|------------|-----------------|-------------------|
| | | | | | |

- (c) If we apply DCT on X_1 , what are the DCT coefficients in the vector $\Theta_1 = AX_1$?
- (d) Generate another 4×1 vector $\overline{\Theta_1}$ by keeping the first three entries of Θ_1 unchanged, while setting its 4th entry to zero. After applying the inverse DCT, what are the entries of the reconstructed vector $\overline{X_1} = A^{-1}\overline{\Theta_1}$?
- (e) What is the value of distortion $|\overline{X_1} \overline{X_1}|^2$? How is the distortion related to the DCT coefficient that was set to zero?
- (f) Fill in the table below with your answers above.

| Θ1 | $\overline{X_1}$ | $ \overline{X_1} - X_1 ^2$ |
|----|------------------|----------------------------|
| | | |

- (g) Attach a screenshot of running your Matlab script.
- (h) Upload the script M file (named "Q4 .m") to Canvas.
- 5. (40 pts) Generation of even and odd images.
 - (a) In Matlab, load a built-in image 'text.png' to the variable I.
 - (b) Display this image.
 - (c) Convert the image to a double-type image f(x,y): >> f = double(I);
 - (d) Write a script to generate three images from f(x,y): f(-x,-y), fe(x,y), and fo(x,y), where fe(x,y) is the even image, and fo(x,y) is the odd image.
 - (e) Check if f(x,y) = fe(x,y) + fo(x,y) by using the *isequal* function.
 - (f) Using the *subplot* function to display the four images in the following configuration:



Use the *imagesc* function to display each of the images. Show the default *colorbar* for each of the images.

(g) Upload the script M file (named "Q5.m") to Canvas.