Homework 4

(Total 200 pts)

Due 5:00 pm, December 6, 2024 (Friday)

Canvas submission as a single PDF file ('hw4_lastname.pdf'),
along with the Matlab scripts (Q1.m, Q2.m, Q4.m, and Q5.m)

1. (50 pts) Revisit Question 2 in HW 3: Mini-project on blending images to create a composite image by superimposing a portrait on a blurred background.
   The smoothing filter will be implemented this time in the frequency domain using the Gaussian low-pass filter (LPF). Use zero padding before DFT domain operations. Compare the various blurring effects by using three distinct standard deviation σ values (1%, 5% and 10% of the padded image size) for the Gaussian filter.
   Note: if the background image is a color image, then apply the same filter on each of the three color channels (R, G, and B) separately, in order to generate the low-pass filtered image in color. Also, you might want to convert the blurred image to uint8 type before overlaying the portrait on it.
   (a) Display the DFT spectrum (centered, and using log function to enhance the contrast) of the background image). If the image is a color image, display the DFT magnitude spectrum for each of the three color components (channels).
   (b) Display the mesh plots for the Gaussian LPF transfer functions $H(u, v)$ (centered) for different σ values.
   (c) Display the composite images generated for different σ values.
   (d) Upload the Matlab scripts you used (named "Q1.m") to Canvas.

2. (40 pts) In Matlab, load the built-in image "coins.png" to the variable f.
   (a) Display the image f.
   (b) Display the centered DFT spectrum $|F(u, v)|$ (use log function to enhance the contrast).
   (c) Given the following 3×3 filter h in the spatial domain, display the filter in the frequency domain using **freqz2** as a mesh plot (by turning the axis off).

$$h = \begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{2}{3} & -\frac{10}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}.$$

   (d) Display $|H|$ as an image, where H is the frequency-domain representation of the filter h after zero-padding according to the size of image f. Center the $|H|$ image.
   (e) Display $|H|$ as an image, but this time the origin is at top left of the frequency rectangle.
   (f) Filter the image f in the spatial domain by h. Zero-pad the image with 0s before filtering. Display the filtering result in spatial domain g_s as an image.
   (g) Filter the image f in the frequency domain by H. Display the filtering result in spatial domain g_f as an image.
   (h) Calculate the largest and smallest values of the absolute difference image $|g_f - g_s|$.
   (i) Attach the Matlab script and the images displayed.
   (j) Attach a screenshot of running your Matlab script to obtain the results above.

(k) Upload the script M file (named "Q2.m") to Canvas.

3. (20 pts) Energy Conservation for Unitary Transforms.
Suppose an $N \times N$ matrix $\mathbf{A}$ has the property that $\mathbf{A}^T\mathbf{A} = \mathbf{A}\mathbf{A}^T = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. If X_1 and X_2 are two N-dimensional column vectors and
$\Theta_1 = \mathbf{A}X_1$
$\Theta_2 = \mathbf{A}X_2$
Show analytically that $|X_1 - X_2|^2 = |\Theta_1 - \Theta_2|^2$, where $|Y|$ denotes the L-2 norm (also called Euclidean length) of a N-dimensional vector Y.

4. (50 pts) Lossy Signal Compression using DCT and Distortions.
(a) In Matlab, run "A = dctmtx (4)" to generate the 4×4 Discrete Cosine Transform (DCT) matrix. What are the entries of the matrix A?
(b) Suppose $X_1 = [1\ 2\ 3\ 4]^T$ and $X_2 = [1\ 1\ 1\ 1]^T$. Fill in the following table with the values of the quantities:

$\|X_1\|^2$	$\|AX_1\|^2$	$\|X_2\|^2$	$\|AX_2\|^2$	$\|X_1 - X_2\|^2$	$\|AX_1 - AX_2\|^2$

(c) If we apply DCT on X_1, what are the DCT coefficients in the vector $\Theta_1 = AX_1$?
(d) Generate another 4×1 vector $\overline{\Theta_1}$ by keeping the first three entries of Θ_1 unchanged, while setting its 4th entry to zero. After applying the inverse DCT, what are the entries of the reconstructed vector $\overline{X_1} = A^{-1}\overline{\Theta_1}$?
(e) What is the value of distortion $|\overline{X_1} - X_1|^2$? How is the distortion related to the DCT coefficient that was set to zero?
(f) Fill in the table below with your answers above.

Θ_1	$\overline{X_1}$	$\|\overline{X_1} - X_1\|^2$

(g) Attach a screenshot of running your Matlab script.
(h) Upload the script M file (named "Q4 .m") to Canvas.

5. (40 pts) Generation of even and odd images.
(a) In Matlab, load a built-in image 'text.png' to the variable I.
(b) Display this image.
(c) Convert the image to a double-type image f(x,y): >> f = double(I);
(d) Write a script to generate three images from f(x,y): f(-x,-y), fe(x,y), and fo(x,y), where fe(x,y) is the even image, and fo(x,y) is the odd image.
(e) Check if f(x,y) = fe(x,y) + fo(x,y) by using the *isequal* function.
(f) Using the *subplot* function to display the four images in the following configuration:

f(x,y)	f(-x,-y)
fe(x,y)	fo(x,y)

Use the *imagesc* function to display each of the images. Show the default *colorbar* for each of the images.
(g) Upload the script M file (named "Q5.m") to Canvas.