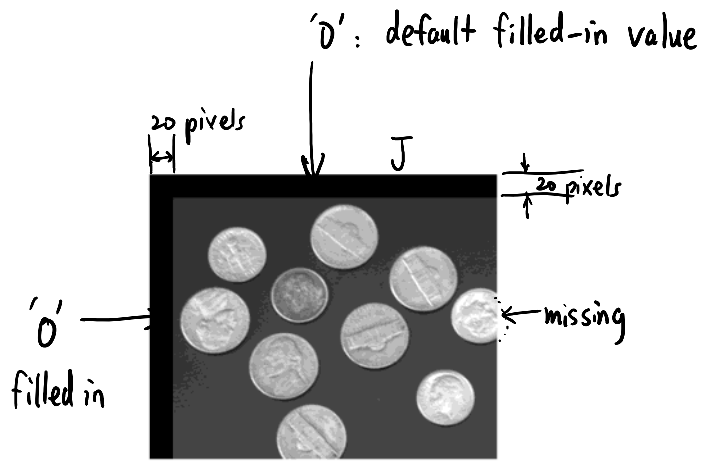
Lecture 10

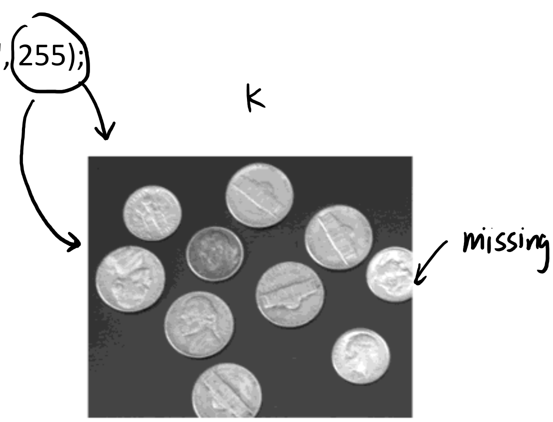
- Fill Values for geometrically transformed images:

```
>> I = imread('coins.png');
>> J = imtranslate(I, [20, 20]);
>> imshow(J)
>> figure; imshow(I)
```

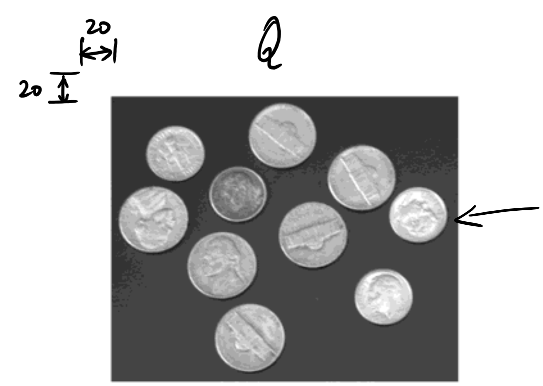



```
>> K = imtranslate(I, [20, 20], 'FillValues', 255);
>> figure; imshow(K)
```

Name	Value
I	246x300 uint8
J	246x300 uint8
K	246x300 uint8




```
>> Q = imtranslate(I, [20, 20], 'FillValues', 255, 'OutputView', 'full');
>> figure; imshow(Q)
```



Q $\quad \underbrace{266}_{246+20}\text{x}\underbrace{320}_{300+20}$ $\quad$ 85120 uint8

Histogram Equalization

Review of Probability Theory

$$f_X(x) = \lim_{e \to 0} \frac{\overbrace{F_X(x+e) - F_X(x)}^{P[x < X \leq x+e}}{e} = \frac{dF_X(x)}{dx}$$

$$\text{CDF} \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} \text{PDF}$$

$$Y = g(X)$$

- Given the PDF of X is known as $f_X(x)$, find the PDF of Y, which is denoted by $f_Y(y)$.
- It is clear that whenever the random variable X lies between x and $x + dx$, the random variable Y will lie between y and $y + dy$.
- Since the probabilities of these events are $f_X(x)dx$ and $f_Y(y)dy$, $f_X(x)dx = f_Y(y)dy$.
- Therefore, $f_Y(y) = f_X(x)\frac{dx}{dy}$.
- In general, $\boxed{f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|}$

Random ( ) in Matlab
Random numbers

'Exponential'	Exponential Distribution	μ mean

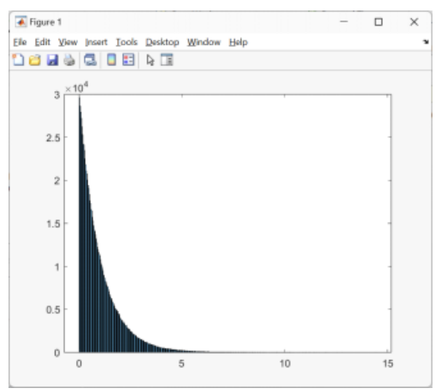
```
>> X = random('exponential',1,1,1000000);
>> doc random
```

Mean $=1$

1X1000000 random numbers

```
>> mean(X)
ans =
   0.9985
```

≈

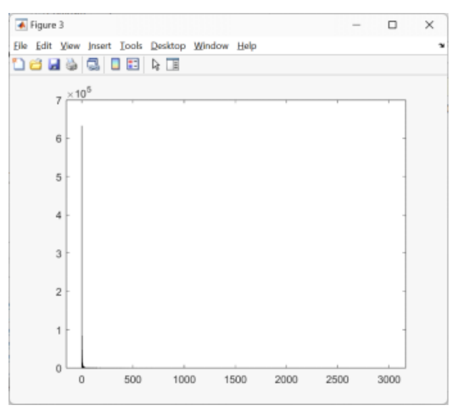


>> histogram(X)

$\int_{x_1}^{x_2} f_X(x)dx = \Pr(x_1 < X \leq x_2)$

$x_1 + e$

```
>> Y = X.^3;
```



>> figure; histogram(Y)

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

$$y = x^3 \Rightarrow \begin{cases} \dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dx}{dy} = \dfrac{1}{3x^2} = \dfrac{1}{3 \cdot y^{\frac{2}{3}}} \\ x = y^{\frac{1}{3}} \end{cases}$$

$$\text{Given } f_X(x) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$$

$$f_Y(y) = \begin{cases} 0, & y < 0 \\ e^{-y^{\frac{1}{3}}} \dfrac{1}{3 \cdot y^{\frac{2}{3}}}, & y > 0 \end{cases}$$

Determine the mapping function for histogram equalization

Continuous Intensity Values

$$\boxed{s = T(r) = (L-1)\int_0^r p_r(\omega)d\omega}$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$= (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right]$$

$$= (L-1)p_r(r)$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

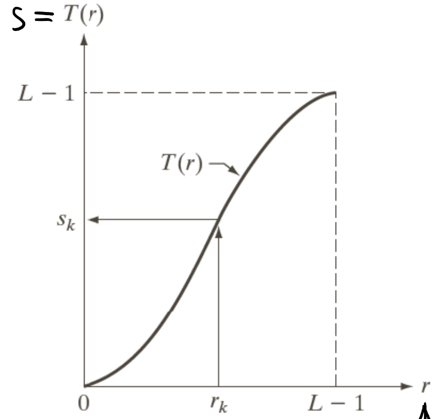
$$= p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right|$$

$$= \frac{1}{L-1} \qquad 0 \le s \le L-1$$

Histogram Equalization

Output Image pixel value

$s = T(r)$



Input Image pixel value