Lecture 17

Post-Test Review (cont'd):

Q5:

(a) $(10)_{10} \rightarrow (00001010)_2 \rightarrow (10)_{10}$

$\uparrow$ LSB $\Leftarrow 0$

$(15)_{10} \rightarrow (00001111)_2 \rightarrow (14)_{10}$

$\uparrow$ LSB $\Leftarrow 0$

$$k(x,y) = \begin{bmatrix} 10 & 14 & 12 \\ 4 & 6 & 10 \\ 8 & 6 & 10 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 255 & 0 \\ 255 & 0 & 255 \\ 0 & 255 & 0 \end{bmatrix}$$

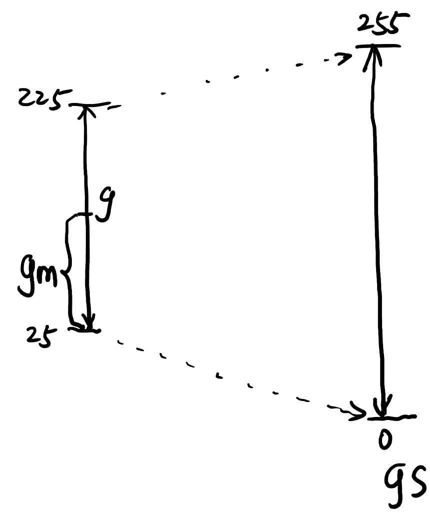
(c)

max

$$\begin{bmatrix} 100 & 225 & 144 \\ 25 & 36 & 121 \\ 64 & 49 & 100 \end{bmatrix}$$

min

any g value



$$\text{round}\left(\frac{g-25}{225-25} \times 255\right)$$

$$\Rightarrow \begin{bmatrix} 96 & 255 & 152 \\ 0 & 14 & 122 \\ 50 & 31 & 96 \end{bmatrix}$$

(d)

$$\begin{matrix} 10 & 10 & 15 & 12 & 12 \\ 10 & 10 & 15 & 12 & 12 \\ 5 & 5 & 6 & 11 & 11 \\ 8 & 8 & 7 & 10 & 10 \\ 8 & 8 & 7 & 10 & 10 \end{matrix}$$

(e)

$$\begin{bmatrix} 86 & 96 & 106 \\ 74 & 84 & 94 \\ 62 & 72 & 82 \end{bmatrix}$$

(f)

$$\begin{bmatrix} 10 & 11 & 12 \\ 8 & 10 & 11 \\ 7 & 8 & 10 \end{bmatrix}$$

Cont'd :

Impulse Noise Analysis (due to noisy communication links, or due to noisy sensors ….)

In general, for the input (original) image I, with pixel value

$$X = \sum_{i=0}^{B-1} \underbrace{b_i}_{\text{bit value}} 2^i, \quad \text{where } B=8$$

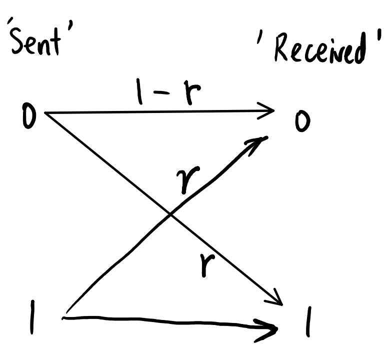
bit value $b_i = \begin{cases} 0 \\ 1 \end{cases}$, where $i = 0, 1, \cdots, B-1$

J : Received image, with pixel value Y

$$\text{Prob}\left[\ |X - Y| = 2^i = ?\ \right]$$

Assume that only the ith-bit was flipped.

BSC :



$P[R1|S1] = 1 - r$

$$\overbrace{\text{Cross-Over}}^{\text{bit being flipped}} \text{ Probability (conditional probabilities)}$$

$r = P[R1|S0]$ ← Condition

$= P[R0|S1]$

Bernoulli Trials:

Example : Flip a $\underbrace{\text{biased}}$ coin 8 times. What is the probability $\underbrace{[\text{Exactly 3 Heads}]}_{\binom{8}{3}\cdot\left(\frac{3}{4}\right)^3\cdot\left(1-\frac{3}{4}\right)^{8-3}}$?

$$P[\underbrace{\text{Head}}_{H}] = \frac{3}{4}, \quad P[\text{Tail}] = \frac{1}{4}$$

$\binom{8}{3}$ Combinations

$$\begin{cases} \text{HHHTTTTT} : & \text{Prob} = \left(\frac{3}{4}\right)^3 \cdot \left(1 - \frac{3}{4}\right)^5 \\ \text{HHTHTTTT} : & \text{Prob} = \left(\frac{3}{4}\right)^3 \cdot \left(1 - \frac{3}{4}\right)^5 \\ \vdots & \vdots \\ \text{TTTTTHHH} : & \text{Prob} = \left(\frac{3}{4}\right)^3 \cdot \left(1 - \frac{3}{4}\right)^5 \end{cases}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}, \qquad \binom{8}{3} = \frac{8!}{3!\,5!} = \frac{8\times7\times6\times5!}{(3\times2)\times5!} = 56$$

Go back to the BSC channel model:

$$\text{Prob}[\text{Only the MSB was flipped}] = r\cdot(1-r)^7$$

previously

Difference between a pixel value before and after its bits go through the channel (BSC)
First, look at the case where only 1 bit was flipped:

MSB: 0 -> 1, difference = (1 - 0) x 128 = 128 $= 2^7$
1 -> 0 , difference = (0 - 1) x 128 = -128
Squared Error (SE) = $(2^7)^2 = 2^{14}$

LSB: 0 ->1, difference = (1 - 0) = 1
1 -> 0 , difference = (0 - 1) = -1
Squared Error (SE) = $(2^0)^2 = 1$

Look at the SE (when only the MSB was flipped)

$$\Downarrow$$

$$(2^{B-1})^2 \cdot \text{Prob}[\text{Only the MSB was flipped}]$$

Compare with the total SE of all other bits being flipped (except for the MSB):

Let $B=8$,

$$\text{Total SE} = (2^0)^2 + (2^1)^2 + (2^2)^2 + \cdots + (2^6)^2 = 4^0+4^1+4^2+\cdots+4^6 = \frac{1-4^7}{1-4} = 5{,}461$$

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>> (4^7 -1)/3
ans = 5461
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In general, for any B

$$\text{Total SE} = 4^0+4^1+\cdots+4^{B-2} = \sum_{i=0}^{B-2} 4^i = \frac{4^{B-1}-1}{3}$$

$$\text{SE}(\text{Only the MSB was flipped}) = 4^{B-1} \gg \frac{4^{B-1}-1}{3}$$

e.g., $B=8$, $4^{8-1} = 16{,}384$

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>> 4^7
ans = 16384
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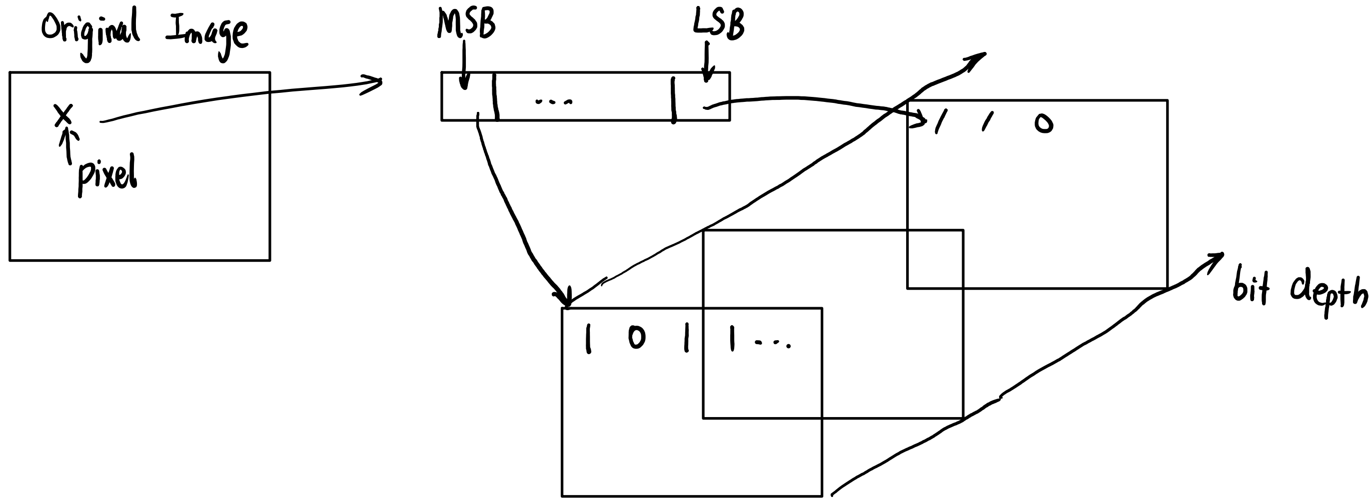
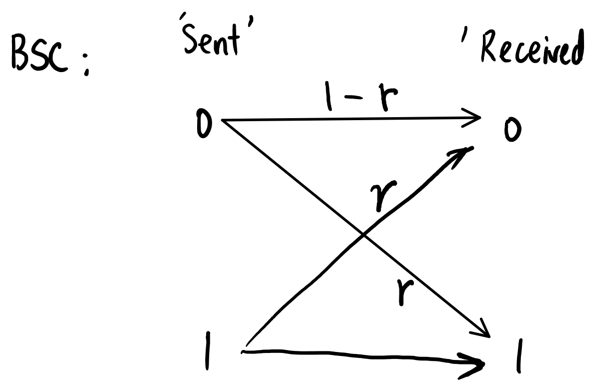
Simulation of BSC on Matlab





$$P[R1|S1] = 1 - r$$