Lecture 20

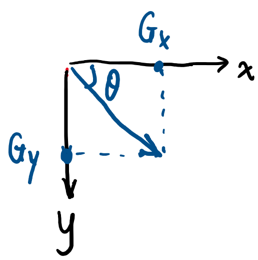
Find directional gradients of 2-D images (cont'd)

```
>> I = imread('Fig0342(a)(contact_lens_original).tif');
>> [Gx, Gy] = imgradientxy(I, 'sobel');
>> [Gmag, Gdir] = imgradient(I);
```

```
>>  Gdir(729, 794)

ans =

 -39.2052
```

```
>> Gx(729, 794)

ans =

   445
```

```
>> Gy(729, 794)

ans =

   363
```



$$\theta = -\operatorname{atan}\left(\frac{G_y}{G_x}\right)$$

```
>> -atan(Gy(729, 794)/Gx(729, 794))*180/pi

ans =

 -39.2052
```

Filtering in the Frequency Domain

http://www.ece.uah.edu/~dwpan/course/ee604/slides/Filtering%20in%20the%20Frequency%20Domain.pdf

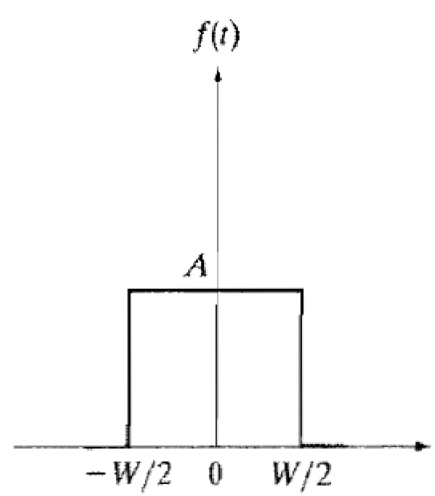
Fourier Transform

$$F(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}\,dt = \int_{-W/2}^{W/2} Ae^{-j2\pi\mu t}\,dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_{-W/2}^{W/2} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right]$$

$$= \frac{A}{j2\pi\mu}\underbrace{\left[e^{\overbrace{j\pi\mu W}^{\theta}} - e^{-j\pi\mu W}\right]}_{\textstyle \to\ e^{j\theta} - e^{-j\theta}}$$

$$= AW\frac{\sin(\pi\mu W)}{}$$



$-W/2 \quad 0 \quad W/2$

$$= AW \frac{\sin(\pi\mu W)}{(\pi\mu W)}$$

$$e^{j\theta} - e^{-j\theta}$$

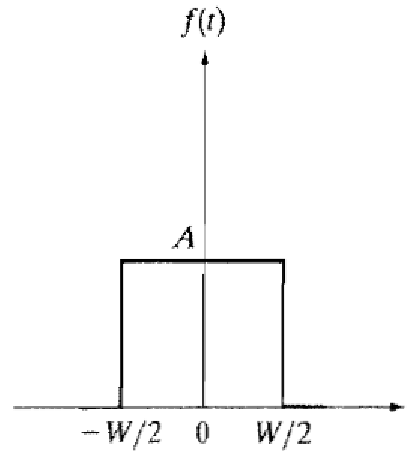
$$= \cos\theta + j\sin\theta - (\cos\theta - j\sin\theta)$$

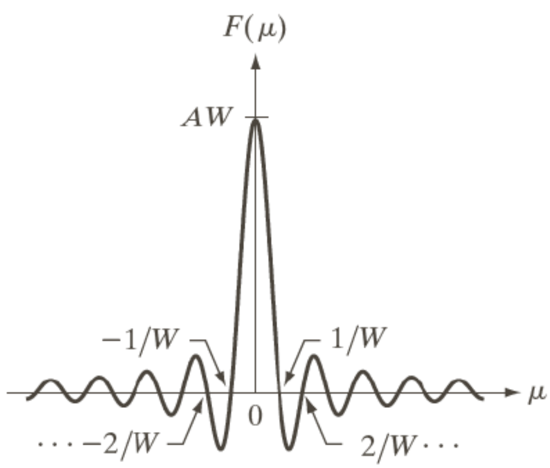
$$= 2j\sin\theta$$

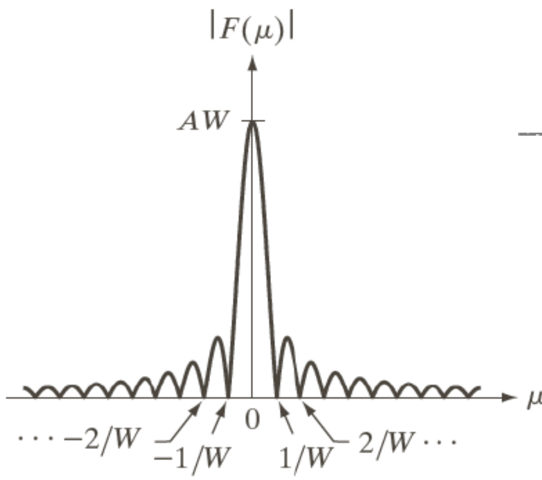
$$\underset{\mu=0}{F(0)} = AW \frac{\sin(0)}{(0)} = AW$$

$$\Im\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt$$

$$F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt \quad \Rightarrow \quad F(0) = \int_{-\infty}^{\infty} f(t)\, dt$$







Discrete Fourier Transform (DFT)

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M} \qquad m = 0, 1, 2, \ldots, M-1$$

digital image (1D):

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \qquad u = 0, 1, 2, \ldots, M-1$$

$$f(x) \xrightarrow{DFT} F(u)$$

$$F(u) = F(u + kM)$$

$$F(u+kM) = \sum f(x)\, e^{-j\frac{2\pi ux}{M} - j\frac{2\pi kMx}{M}} = \sum f(x)\, e^{-j\frac{2\pi ux}{M}} \cdot \underbrace{e^{-j\frac{2\pi kMx}{M}}}_{1}$$

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u)\, e^{j2\pi ux/M} \qquad x = 0, 1, 2, \ldots, M-1$$

$$f(x) = f(x + kM)$$

fft
Fast Fourier transform

```
>> f = [1 2 4 4];
>> f
f =
    1    2    4    4
```

f(0) f(1) f(2) f(3)

```
>> fft(f)

ans =
  11.0000 + 0.0000i  -3.0000 + 2.0000i  -1.0000 + 0.0000i  -3.0000 - 2.0000i
```

F(0) F(1) F(2) F(3)

$$F(0) = \sum_{x=0}^{3} f(x) = \left[f(0) + f(1) + f(2) + f(3)\right] = 1 + 2 + 4 + 4 = 11$$

$$F(1) = \sum_{x=0}^{3} f(x) e^{-j2\pi(1)x/4}$$

$$= 1e^{0} + 2e^{-j\pi/2} + 4e^{-j\pi} + 4e^{-j3\pi/2} = -3 + 2j$$

IDFT :

$$f(0) = \frac{1}{4}\sum_{u=0}^{3} F(u)\, e^{j2\pi u(0)}$$

$$= \frac{1}{4}\sum_{u=0}^{3} F(u) = \frac{1}{4}[11 - 3 + 2j - 1 - 3 - 2j] = \frac{1}{4}[4] = 1$$

```
>> F = fft(f);
>> ifft(F)

ans =

    1    2    4    4
```

2-D DFT and its Properties

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Periodicity

$$F(u, v) = F(u + k_1 M, v) = F(u, v + k_2 N) = F(u + k_1 M, v + k_2 N)$$

$$f(x, y) = f(x + k_1 M, y) = f(x, y + k_2 N) = f(x + k_1 M, y + k_2 N)$$