Lecture 22

DFT Spectrum Shifting

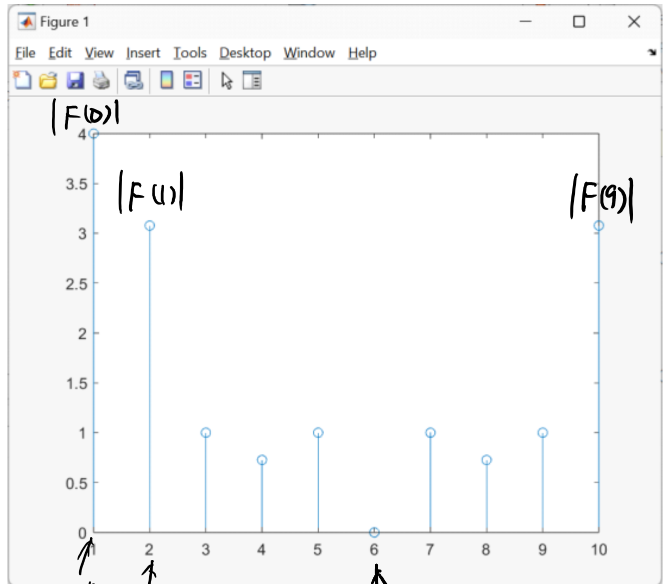
```
>> f = [0 0 0 1 1 1 1 0 0 0];
>> F = fft(f)

F =
  Columns 1 through 3
   4.0000 + 0.0000i  -2.9271 - 0.9511i   0.8090 + 0.5878i

  Columns 4 through 6
   0.4271 + 0.5878i  -0.3090 - 0.9511i   0.0000 + 0.0000i
  Columns 7 through 9
  -0.3090 + 0.9511i   0.4271 - 0.5878i   0.8090 - 0.5878i
  Column 10
  -2.9271 + 0.9511i
```

$F(1)$

$|F(9)| = |F(1)| = |F(10-9)|$



$M=10$

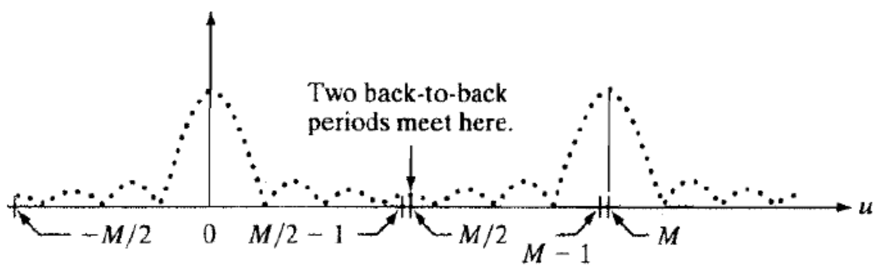
>> stem(abs(F))

$|F(\frac{M}{2})| = |F(5)|$

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \qquad u = 0, 1, 2, \ldots, M-1$$

$|F(u)| = |F(M-u)|$?

Try to prove this.



Next

$$F\left(u-\frac{M}{2}\right) \underset{\text{DFT}}{\overset{\text{DFT}^{-1}}{\rightleftarrows}} f(x)(-1)^x$$

$$= \sum_{x=0}^{M-1} f(x)\, e^{-j2\pi\left(u-\frac{M}{2}\right)x/M} = \sum_{x=0}^{M-1} f(x)\, e^{-j2\pi ux/M} \cdot \underbrace{e^{j\pi x}}_{(e^{j\pi})^x = (-1)^x} = \sum_{x=0}^{M-1} \left[f(x)(-1)^x\right] \cdot e^{-j\frac{2\pi ux}{M}}$$

$$F\left(u-\frac{M}{2}\right) \underset{DFT}{\overset{DFT^{-1}}{\rightleftarrows}} f(x)(-1)^x$$

$\downarrow$

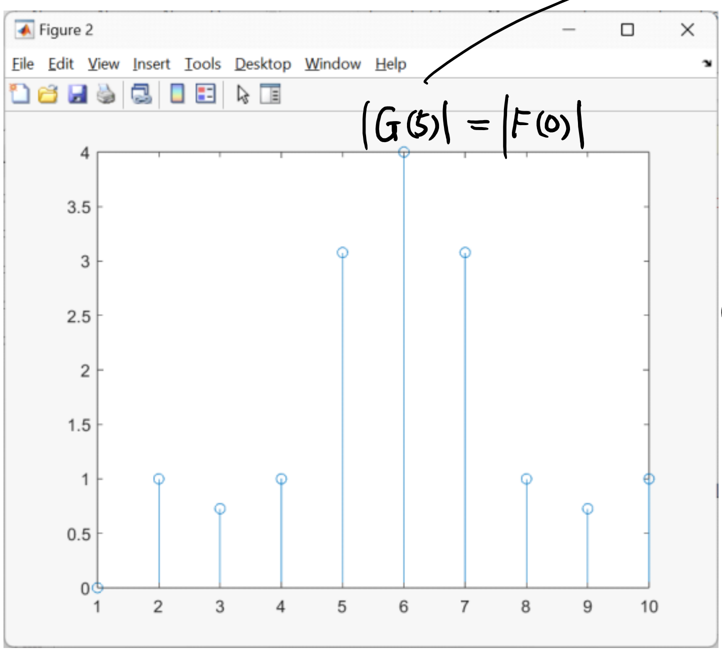
If x is an even number: $f(x)\cdot(-1)^x = f(x)$

If x is an odd number: $f(x)\cdot(-1)^x = -f(x)$

```
>> f = [0 0 0 1 1 1 1 0 0 0];
>> sign = [1 -1 1 -1 1 -1 1 -1 1 -1];
>> g = f.*sign
g =
   0    0    0   -1    1   -1    1    0    0    0
```

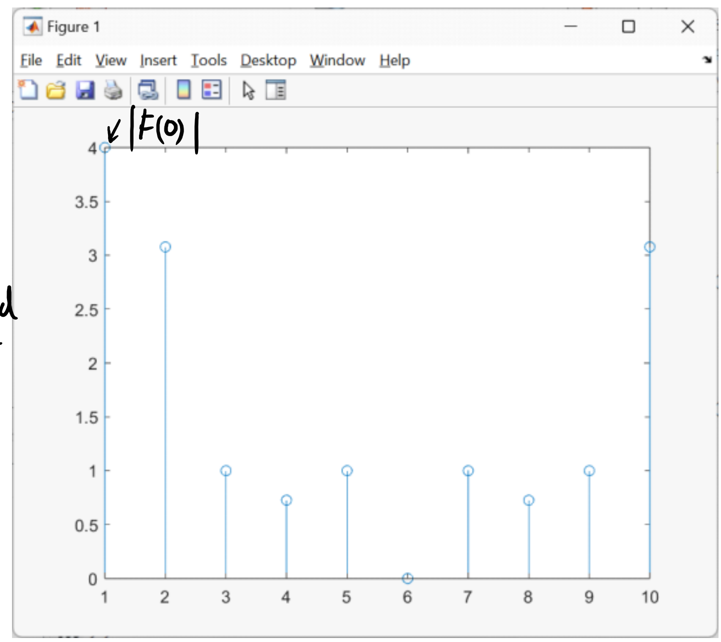
$G(5) = F\left(5-\frac{10}{2}\right) = F(0)$

$|G(5)| = |F(0)|$



Centered $\leftarrow$

$|F(0)|$




```
>> G = fft(g);
>> figure; stem(abs(G))
```

Alternatively, use
fftshift in Matlab

fftshift
Shift zero-frequency component to center of spectrum

```
>> H = fftshift(F);
>> isequal(G, H)
ans =
  logical
   1
```

```
g = f.*sign
G = fft(g);
```

$(-1)^1 \cdot 2$

Another simpler example:

1

Another simpler example:

```
>> A = [1 2]
>> F = fft(A)
F =
   3   -1
```

$(-1)^1 \cdot 2$

```
>> B = [1 -2];
>> fft(B)
ans =
  -1    3
```

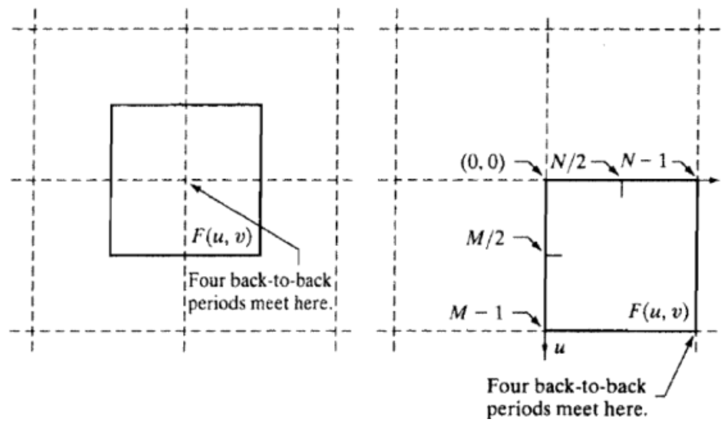
Same result

```
>> fftshift(F)
ans =
  -1    3
```

2-D DFT Centered Spectrum

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

$$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$$




```
>> A = [1 2; 3 4]
A =
     1     2
     3     4

>> F = fft2(A)
F =
    10    -2
    -4     0

>> fftshift(F)
ans =
```

$$\begin{bmatrix} 0 & -4 \\ -2 & 10 \end{bmatrix}$$

Same result

```
>> B = [1 -2; -3 4]

B =
```

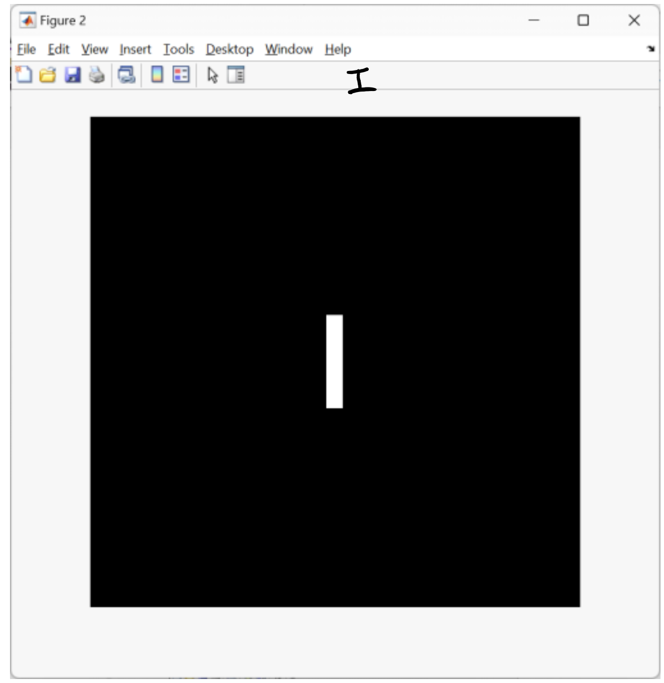
$$\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{A} \cdot (-1)^{x+y} = \begin{bmatrix} 1 \cdot (-1)^{0+0} & 2 \cdot (-1)^{0+1} \\ 3 \cdot (-1)^{1+0} & 4 \cdot (-1)^{1+1} \end{bmatrix}$$

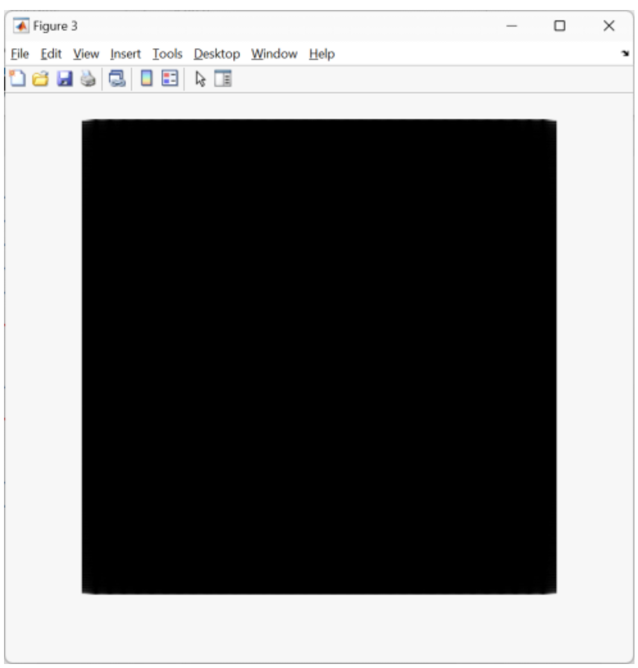
```
>> fft2(B)
ans =
```

$$\begin{bmatrix} 0 & -4 \\ -2 & 10 \end{bmatrix}$$

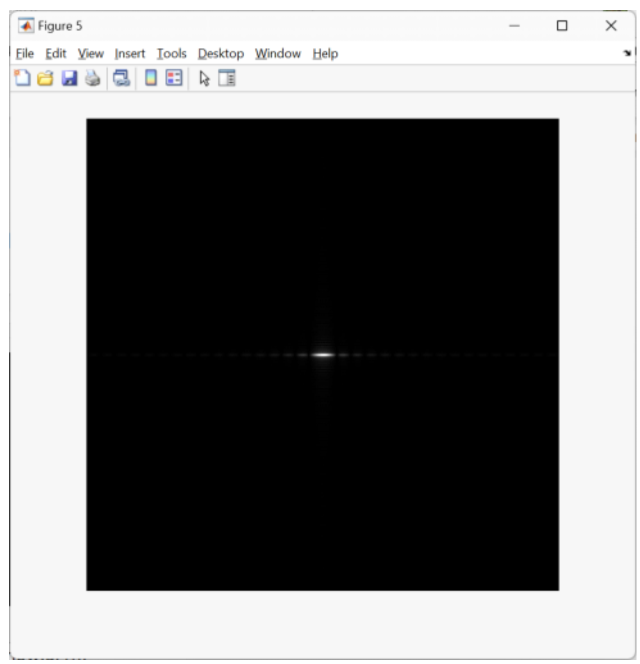
```
>> I = imread('Fig0424(a)(rectangle).tif');
>> imshow(I)
>> F = fft2(I);
>> S = abs(F);
>> max(S(:))
ans =
    1681980

>> min(S(:))
ans =
    0
```

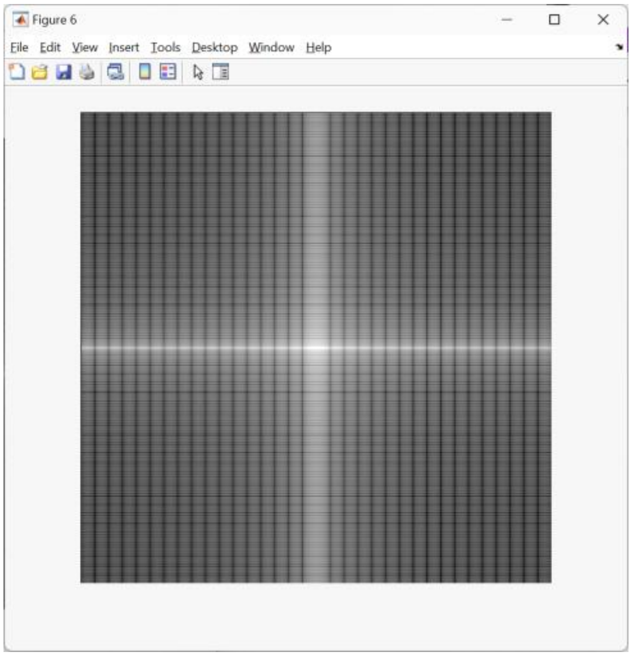





```
>> figure; imshow(S, []);
```




```
>> Fc = fftshift(F);
>> figure; imshow(abs(Fc), []);
```




```
>> S2 = log(1 + abs(Fc));
>> figure; imshow(S2, [])
```