

# Lecture 23

ifftshift

Inverse zero-frequency shift

A =

1 2

3 4

>> fftshift(A)

ans =

4 3

2 1

>> ifftshift(fftshift(A))

ans =

1 2

3 4

Duality (Spatial vs. Frequency  
Domain Representations)

>> I = imread('Fig0424(a)

(rectangle).tif');

>> imshow(I)

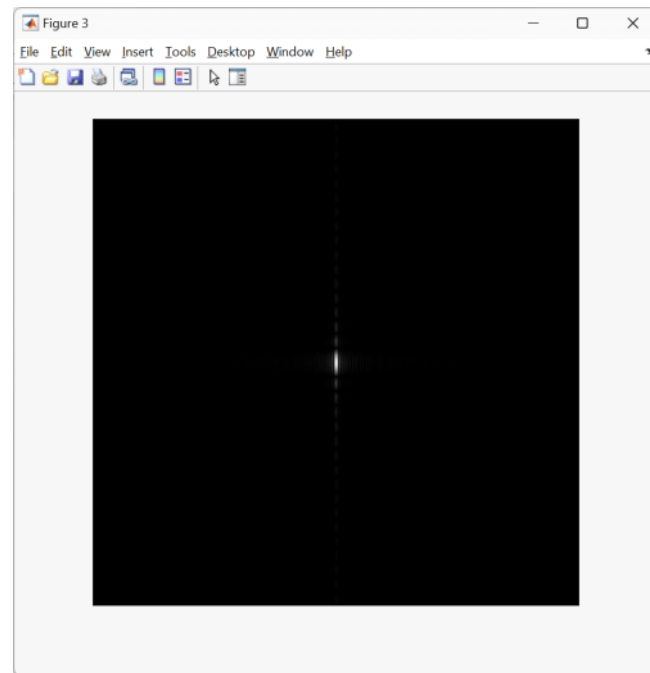
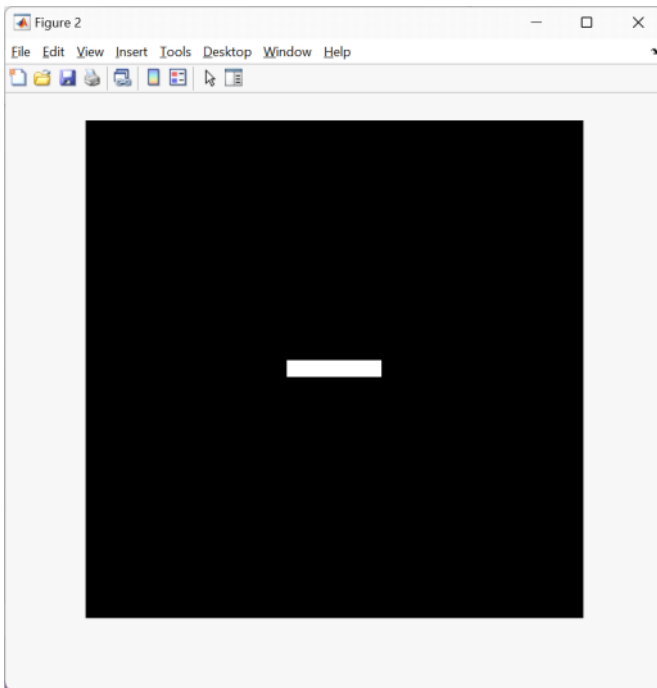
>> J = I';

>> figure; imshow(J)

>> G = fft2(J);

>> figure; imshow(abs(fftshift(G)),[]);

>>

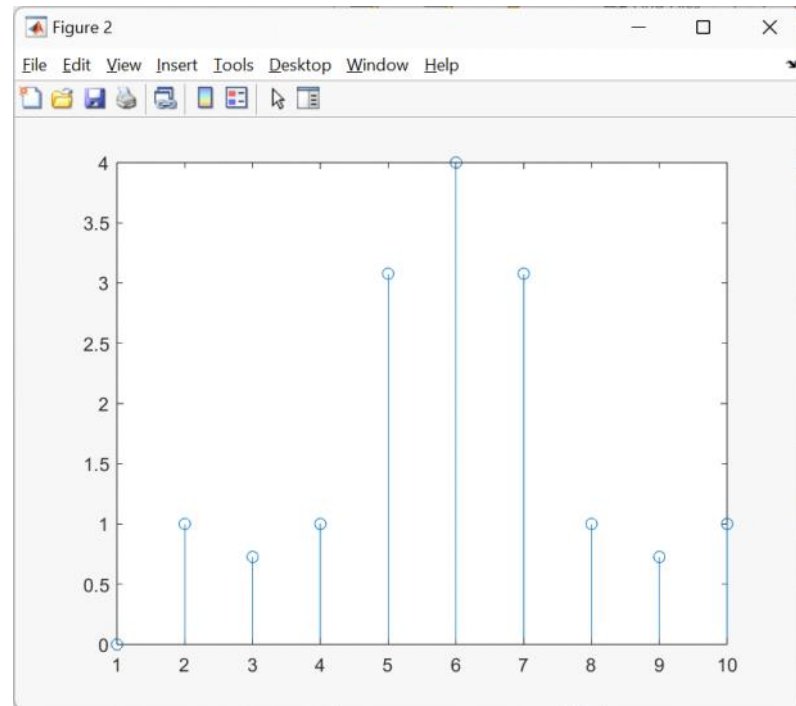


Symmetry of DFT coefficients

f = [0 0 0 1 1 1 1 0 0 0];

$$F(u) = F(u + kM)$$

>> figure; stem(abs(fftshift(F)))



In general, for 2D DFT, if  $f(x,y)$  is real, then  
 $R(u,v) = \text{Re}(F(u,v))$  is an even image;  
 $I(u,v) = \text{Im}(F(u,v))$  is an odd image.

$$f(x, y) \text{ real} \iff R(u, v) \text{ even}; I(u, v) \text{ odd}$$

Quick Review:

- 1D signal:

$f(x) = f_e(x) + f_o(x)$ , where

$f_e(x) = [f(x) + f(-x)]/2$ ;

$f_o(x) = [f(x) - f(-x)]/2$ ;

$$\Rightarrow f_e(-x) = \frac{f(-x) + f(-(-x))}{2} = \frac{f(-x) + f(x)}{2} = f_e(x)$$

$$\Rightarrow f_o(-x) = -f_o(x)$$

Definitions:

Even:  $f_e(x) = f_e(-x)$ ;

Odd:  $f_o(x) = -f_o(-x)$ .

For example, if  $M = 4$

Even Sequence:  $\{2, 1, 1, 1\}$ , periodically extended to  $\{2, 1, 1, 1\}, \{2, 1, 1, 1\}, \{2, 1, 1, 1\}$

?

$$f(0) = 2 = f(0-M) = f(-4) = 2, \quad f(-4) = f(-4+M) = f(4+4) = f(0)$$

$$f(1) = 1 = f(1-M) = f(-3) = f(3) = 1, \quad f(-3) = f(-3+M) = f(-3+4) = f(1)$$

$$f(2) = 1 = f(4-2) = f(2) = 1$$

Odd Sequence:  $\{0, -1, 0, 1\}$ , periodically extended to ...

$$w_e(x, y) = w_e(-x, -y)$$

$$w_o(x, y) = -w_o(-x, -y)$$

$$w_e(x, y) = w_e(M - x, N - y)$$

$$w_o(x, y) = -w_o(M - x, N - y)$$

$M$  and  $N$  are the number of rows and columns of a 2-D array

$$w(x, y) = w_e(x, y) + w_o(x, y)$$

$$w_e(x, y) \triangleq \frac{w(x, y) + w(-x, -y)}{2}$$

$$w_o(x, y) \triangleq \frac{w(x, y) - w(-x, -y)}{2}$$

Question:

$w(-x, -y)$  ?

```
>> M = 4;
>> fm = zeros(M, M);
>> for x = 0: (M-1)
    for y = 0: (M-1)
        fm(x+1, y+1) = f(mod(M-x,M)+1, mod(M-y,M)+1);
    end
end
>> fm
fm =
    16    13     3     2
     4     1    15    14
     9     7     6    12
     5     8    10    11
```

```
>> fe = (f + fm)/2
fe =
    16.0000    7.5000    3.0000    7.5000
     4.5000    6.0000   12.5000   11.0000
     9.0000    9.5000    6.0000    9.5000
     4.5000   11.0000   12.5000    6.0000
```

```
>> f = magic(4)
```

```
f =
```

```
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

```
>> fe+fo
```

```
ans =
```

```
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

```
>> fo = (f - fm)/2
```

```
fo =
```

```
     0  -5.5000     0  5.5000
    0.5000  5.0000  -2.5000  -3.0000
     0  -2.5000     0  2.5000
   -0.5000  3.0000  2.5000  -5.0000
```

4.5000	6.0000	12.5000	11.0000
9.0000	9.5000	6.0000	9.5000
4.5000	11.0000	12.5000	6.0000

0.5000	3.0000	-2.5000	-3.0000
0	-2.5000	0	2.5000
-0.5000	3.0000	2.5000	-5.0000