Homework 3

(Total 120 pts)

Due 11:55 pm on September 30, 2022 (Friday)

Canvas submission of your answers (with required plots and source codes attached) in a single PDF file ('hw3.pdf'), and then submit the following source code files: Q2.m and Q3.py

1. (40 pts) Trace of matrix product.
   Let **A** and **B** be any 3×3 matrices:
   $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, and $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$.
   (a) Show that $tr(\mathbf{AB}) = tr(\mathbf{BA})$.

   (b) Let $f = tr(\mathbf{AB})$, determine the matrix derivatives below:
   $\frac{\partial f}{\partial \mathbf{A}} = \begin{bmatrix} \frac{\partial f}{\partial a_{11}} & \frac{\partial f}{\partial a_{12}} & \frac{\partial f}{\partial a_{13}} \\ \frac{\partial f}{\partial a_{21}} & \frac{\partial f}{\partial a_{22}} & \frac{\partial f}{\partial a_{23}} \\ \frac{\partial f}{\partial a_{31}} & \frac{\partial f}{\partial a_{32}} & \frac{\partial f}{\partial a_{33}} \end{bmatrix}$. Simplify your expression as much as possible.

2. (30 pts) K-Means clustering.
   (a) Read in the following dataset:
   http://www.ece.uah.edu/~dwpan/course/ee610/hw/dataset_hw.csv
   (b) Perform 2-Means cluster analysis on the observation vectors (x_1 and x_2, respectively in the first two columns of the csv file), by using the *kmeans* function in Matlab. Use the default Euclidean distance metric.
   (c) Display the scatter plot of the clusters. Use red color for cluster 1, and blue color for cluster 2. Set 'MarkerSize' to be 12.
   (d) On the same scatter plot, display the centroids in dark color. Also, set 'MarkerSize',15,'LineWidth',3.
   (e) Add proper legends to the plot.
   (f) Fill in the table below with your answers to the result of 2-Means analysis.

	Centroid Location (x_1, x_2)	Within-cluster **average** of point-to-centroid distances (without being squared)	Within-cluster **sum** of point-to-centroid distances (squared)
Cluster Index 1	(,)		
Cluster Index 2	(,)		

   (g) Attach your script and plot.

3. (20 pts) K-Means clustering using sklearn.
   (a) Read in the following dataset:
   http://www.ece.uah.edu/~dwpan/course/ee610/hw/dataset_hw.csv

(b) Perform 2-Means cluster analysis on the observation vectors (x_1 and x_2, respectively in the first two columns of the csv file), by setting "random_state = 0" for the Kmeans API in sklearn.
(c) Display the attribute "cluster_centers_". What are their values? Did you get the same values as those of the centroid locations reported in Q2(f)?
(d) Display the attribute "inertia_" of the trained cluster. What is the value of "inertia_"? How is this value related to the results reported in Q2(f)?
(e) Attach the Python code.

4. (30 pts)
Given a matrix $\boldsymbol{C} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, determine analytically its distinct eigenvalues. Show that the eigenvectors corresponding to the distinct eigenvalues are orthogonal to each other. Show your detailed derivations.