Lecture 13

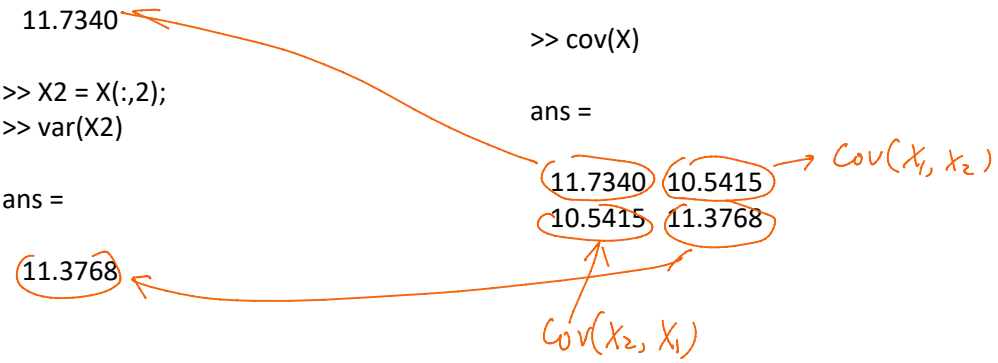
Principal Component Analysis

```
N = 100;

% Generate data entries for Class 1
m1 = [3, 3]';   % Mean vector
cov1 = [2 1; 1 2]; % Covariance matrix
rng default  % For reproducibility
r1 = mvnrnd(m1,cov1,N);

data_C1 = zeros(N, 2);
data_C1 = r1;

% Generate data entries for Class 2
m2 = [9, 9]';
cov2 = [2,1; 1,2];
rng default  % For reproducibility
r2 = mvnrnd(m2,cov2,N);

data_C2 = zeros(N, 2);
data_C2 = r2;

% Combine data of two classes
X = vertcat (data_C1, data_C2);
```

```
>> whos X
  Name        Size              Bytes  Class     Attributes

  X         200x2              3200  double

>> X(1,:)

ans =

    3.7604    4.4094

>> X(2,:)

ans =

    5.5935    3.2091

>> var(r1)

ans =

    2.7023    2.3434

>> X1 = X(:,1);
>> whos X1
  Name        Size              Bytes  Class     Attributes

  X1        200x1              1600  double
```

```
>> var(X1)

ans =

   11.7340

>> X2 = X(:,2);
>> var(X2)

ans =

   11.3768
```

```
>> cov(X)

ans =

   11.7340   10.5415
   10.5415   11.3768
```



C — Covariance Matrix

$C = C^T$

$C V_1 = \lambda_1 V_1$ (eigen value)

$(m \times m)(m \times 1) \rightarrow (m \times 1)$ (λ_1: scalar)

$m \times 1$

$C V_2 = \lambda_2 V_2$

$V_1^T V_2 = 0 \Rightarrow V_1^T (2V_2) = 0$

```
>> mu = mean(X);
>> X_mr = X - mu;
>> C = cov(X_mr)

C =

  11.7340   10.5415
  10.5415   11.3768

>> [V,D] = eig(C);
>> V

V =

   0.7011   -0.7131
  -0.7131   -0.7011

>> D

D =

   1.0124         0
        0   22.0984

>> V1 = V(1,:)

V1 =

   0.7011   -0.7131

>> C*V1'

ans =

    0.7098
   -0.7219

>> D(1,1)*V1'

ans =

    0.7098
   -0.7219

>> norm(V1)

ans =

    1.0000

>> norm(V2)

ans =

    1.0000

>> V2 = V(2,:)'

V2 =

   -0.7131
   -0.7011

>> C*V2

ans =

  -15.7577
  -15.4930

>> D(2,2)*V2

ans =

  -15.7577
  -15.4930

>> V1 = V1'

V1 =

    0.7011
   -0.7131

>> V2

V2 =

   -0.7131
   -0.7011

>> V1'*V2

ans =

     0
```

SVD

$$X = ASB^T \Rightarrow X^T = (ASB^T)^T = (B^T)^T S^T A^T = BS^T A^T$$

$$X^T X = BS^T \underbrace{A^T \cdot A}_{I} SB^T = B \underbrace{S^T \cdot S}_{\Sigma} B^T = B\Sigma B^T \quad \cdots 1)$$

where $A^T \cdot A = I$

and $B^T \cdot B = I$

Σ: diagonal matrix

>> S(1:4, 1:2)

ans =

138.6237 0
0 14.1984
0 0
0 0
0 0
...

$$X = A \cdot S \cdot B^T \quad , \quad S^T \cdot S = \Sigma$$

$N \times m$ $(N \times N)$ $(N \times m)$ $(m \times m)$ $(m \times N)$ $(N \times m)$ $m \times m$

200×2 200×2 2×2

$B: (m \times m)$

From Eq. 1)

$$(X^T X) B = B \Sigma \underbrace{B^T \cdot B}_{I} = B\Sigma$$

$(m \times N)(N \times m)$ $(m \times m)$

B: eigenvector matrix of $X^T X$

>> B'*B

ans =

1.0000 0.0000
0.0000 1.0000

>> X'*X*B

ans =

1.0e+04 *

-1.3766 -0.0141
-1.3408 0.0144

>> sigma = S'*S

sigma =

1.0e+04 *

1.9217 0
0 0.0202

>> B*sigma

ans =

1.0e+04 *

-1.3766 -0.0141
-1.3408 0.0144