Lecture 14

Post-test review

Q1 :

$$f(x) = (Vx-y)^T(Vx-y) = \|Vx-y\|^2$$

$$= 29x_1^2 + 18x_1x_2 - 134x_1 - 42x_2 + 3x_2^2 + 155$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 58x_1 + 18x_2 - 134 \\ 18x_1 + 6x_2 - 42 \end{bmatrix} = Px + Q$$

$$= \begin{bmatrix} 58 & 18 \\ 18 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -134 \\ -42 \end{bmatrix}$$

Alternatively,

$$\nabla f = \nabla_x \left[(Vx-y)^T(Vx-y) \right] = \underbrace{2(V^TV)}_{P}x \underbrace{- 2V^Ty}_{Q}$$

Q2 :

$$d_j(x) = p(x|c_j)P(c_j) = \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x-m_j)^2}{2\sigma_j^2}} P(c_j)$$

where $j = 1, 2$

$$d_1(x) = \frac{1}{\sqrt{2\pi}\cancel{\sqrt{3}}} e^{-\frac{(x-3)^2}{2\cdot 3}} \left(\frac{3}{4}\right)$$

$$d_2(x) = \frac{1}{\sqrt{2\pi}\cancel{\sqrt{3}}} e^{-\frac{(x-8)^2}{2\cdot 3}} \left(\frac{1}{4}\right)$$

$\rightarrow d_1(x) = d_2(x)$

$$-\frac{(x-3)^2}{6} + \ln\left(\frac{3}{4}\right) = -\frac{(x-8)^2}{6} + \ln\frac{1}{4}$$

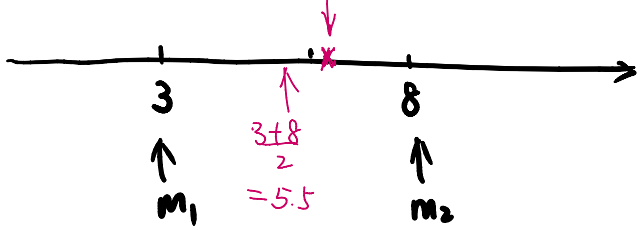
$$-\frac{(x-3)^2}{6}+\ln\left(\frac{3}{4}\right) = -\frac{(x-8)^2}{6}+\ln\frac{1}{4} \quad , \quad -\ln\left(\frac{1}{4}\right)+\ln\left(\frac{3}{4}\right) = \ln\frac{\frac{3}{4}}{\frac{1}{4}} = \ln 3$$

$$\ln 3 - \frac{(x-3)^2}{6} = -\frac{(x-8)^2}{6}$$

$$6\ln 3 - (x-3)^2 = -(x-8)^2$$

$$6\ln 3 = (x-3)^2 - (x-8)^2 = 10x - 55$$

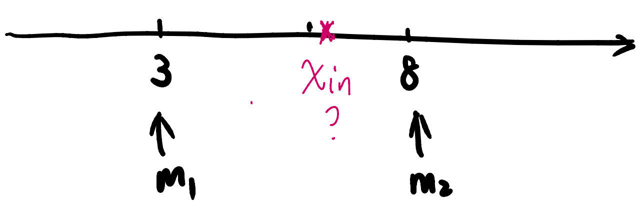
$$x = \frac{6\ln 3 + 55}{10} = 6.1592$$



Q3 :

$$\sqrt{(x_{in}-m_1)\,\sigma_1^{-2}\,(x_{in}-m_1)^T} = \frac{x_{in}-m_1}{\sigma_1}$$

$$= \frac{m_2-x_{in}}{\sigma_2}$$



$$\frac{x_{in}-3}{1} = \frac{8-x_{in}}{2}$$

$$\Rightarrow x_{in} = \frac{14}{3}$$

Q4.

Y:	1	1	1	1	2	2	2	2
Pred:	1	1	2	2	1	2	2	2

$x = 3.0$ ......

3.0	Class 1
4.0	Class 1
4.5	Class 2

6.5

6.5	Class 2
5.5	Class 2
7.0	Class 1

$$\text{Accuracy} = \frac{5}{8}$$

Q5: (a)

$$\text{Recall} = \frac{TP}{P} = \frac{3}{4}$$

$$\text{Specificity} = \frac{TN}{N} = \frac{2}{4} = 0.5$$

(b)

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{2}{3}$$

$$F_1 = \frac{2TP}{2TP + FP + FN} = \frac{4}{7}$$