Lecture 4

Entropy

For a purely random image: $P(k) = \frac{1}{256}$

$$\text{Entropy} = \sum_{k=0}^{255} - P(k) \cdot \log_2 P(k) = -256 \times \left(\frac{1}{256} \times \log_2 \frac{1}{256} \right) = 8 \text{ bits/symbol}$$

Review of Probability theory

Random Variables (RV's): X

CDF: Cumulative Distribution Function

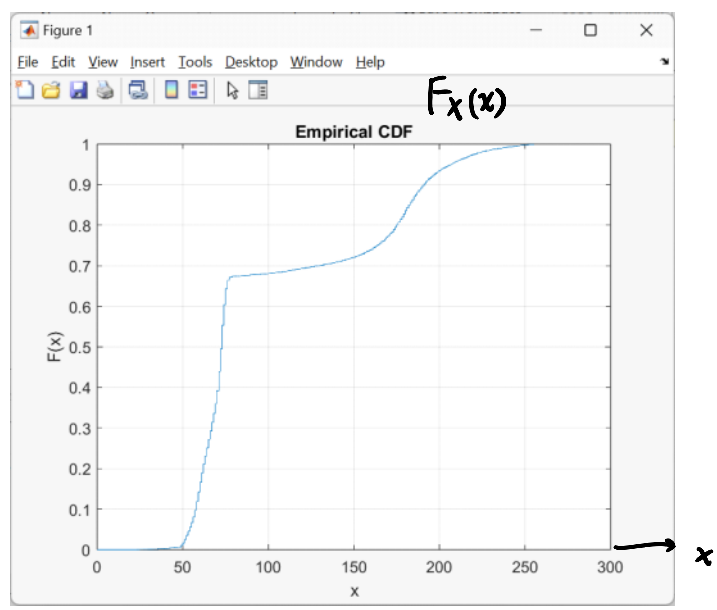
$$\underset{\substack{\uparrow \\ \text{RV}}}{F_X}(x) = P(\underset{\substack{\uparrow \\ \text{RV}}}{X} \leq \underset{\substack{\uparrow \\ \text{any given value}}}{x})$$

```
>> I = imread('coins.png');
>> I1d = reshape(I, 1, 73800);
>> cdfplot(I1d)

>> y = find(I1d <= 149);
>> length(y)/length(I1d)

ans =

   0.720298102981030
```



- Continuous RV's
  PDF (Probability Density Function)

$$f_X(x) = \frac{d F_X(x)}{dx}$$

Discrete RV's
PMF (Probability Mass Function)

$$P(X = x_i)$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\sum_{\text{all } i} P(X = x_i) = 1$$

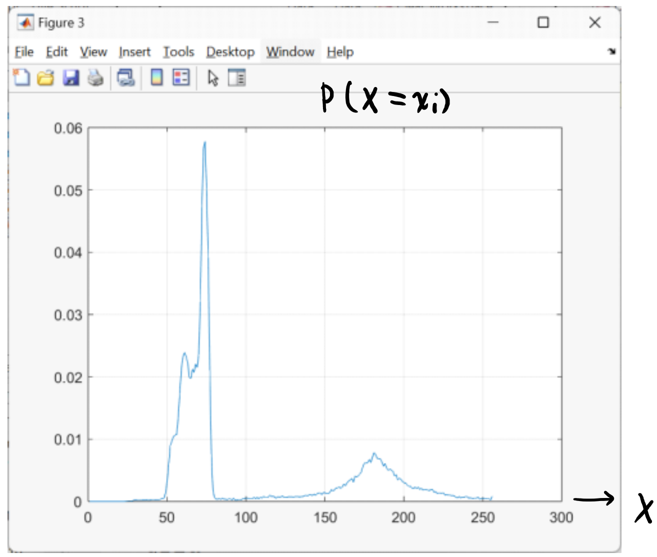
Moments:

Expected Value (Mean) : $E[X] = \sum_{\text{all } i} x_i \cdot P(X = x_i)$, $E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$

```
>> I = imread('coins.png');
>> [c, b] = imhist(I);
>> p = c/(246*300);
>> figure; plot(p)
>> sum(p)
ans =
     1

>> sum(b.*p)
ans =
  102.9791

>> mean(I, "all")
ans =
  102.9791
```
=> same result



- Mean Square

$$E[X^2] = \sum_{\text{all } i} x_i^2 \cdot P(X = x_i)$$

- Expected value of a function on random variables

$$E[g(X)] = \sum_{\text{all } i} g(x_i) \cdot P(X = x_i) , \qquad E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

Example: RV with Geometric Distribution

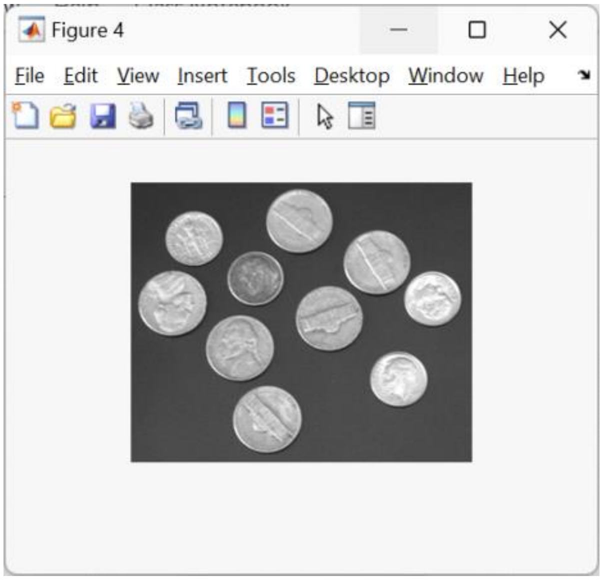
X: takes non-negative integer values (n)

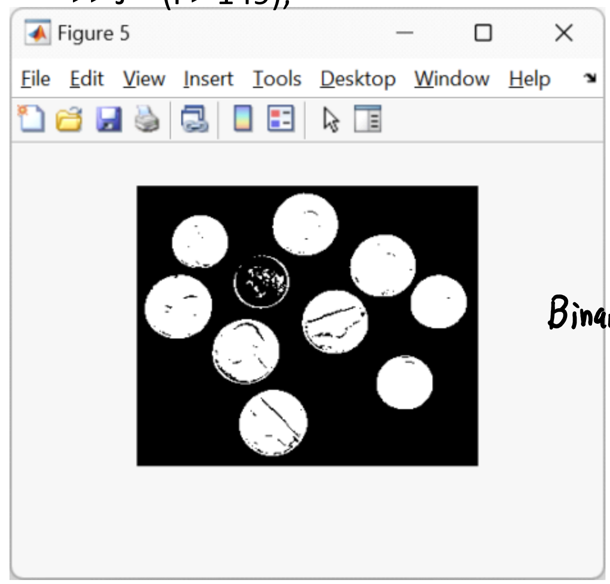
$G(n) = P(X = n) = p^n (1-p)$, where $0 < p < 1$: given parameter

Assume that the probability of a symbol '0' occurring is: $p \Rightarrow P('1') = 1-p$ for binary source

$$P(\underbrace{0\,0 \cdots 0}_{\text{run length} = n}\,1) = p^n \cdot (1-p)$$

```
>> figure; imshow(I)
>> J = (I > 149);
```





Binary Image

```
>> [c, b] = imhist(J);
>> p = c/(246*300);
>> sum(p)
ans =
   1
```

```
>> b
b =
    0
    1
```

```
>> stem(b,p)
```

PMF of the above binary image