Homework 1

(Total 130 pts)

Due 5:00 pm on June 12, 2024 (Wednesday)

Note: You work must be electronically submitted to Canvas as a single **PDF** file.

1. (20 pts) A system has the input-output relation given by
   $y[n] = T\{x[n]\} = nx[n]$.
   Determine whether the system is
   (a) linear; (b) time invariant; (c) stable; (d) causal.

2. (20 pts) Let x[n] = δ[n] + 2δ[n-1] - δ[n-3] and h[n] = 2δ[n+1] + 2δ[n-1]. Compute the following convolution: y[n] = x[n] * h[n].

3. (30 pts)
   (a) Calculate the Fourier transform of $X(e^{j\omega})$ of the sequence:
   $x[n] = u[n + 3] – u[n - 4]$.
   (b) In Matlab, plot and label the magnitude of the Fourier transform $\left|X(e^{j\omega})\right|$ (where ω goes from –2π to 2π. Attach a printed hardcopy of the plot.

4. (20 pts)
   (a) If $x[n]$ is a complex sequence with Fourier transform being $X(e^{j\omega})$, prove that
   $\mathcal{F}\{x^*[-n]\} = X^*(e^{j\omega})$.
   (b) Suppose $x[n]$ is a real sequence. Use Property 7 of the Symmetry of Fourier Transforms to prove that the imaginary part of its Fourier transform is odd, i.e.,
   $X_I(e^{j\omega}) = -X_I(e^{-j\omega})$.

5. (10 pts) Determine the frequency response $H(e^{j\omega})$ of the LTI system whose input and output satisfy the following difference equation.
   $$y[n] + \frac{1}{2}y[n-1] + \frac{3}{4}y[n-2] = x[n] - \frac{1}{2}x[n-1] + x[n-3].$$

6. (10 pts) Suppose the frequency response of an LTI system is given by
   $H(e^{j\omega}) = \dfrac{1}{1-0.5e^{j\omega}}$. If the input to the system is $x[n] = 1 + e^{j\pi n}$, what is the output $y[n]$?

7. (20 pts) The sequences $s[n]$, $x[n]$ and $w[n]$ are sample sequences of wide-sense stationary random processes where $s[n] = x[n]w[n]$. The sequences $x[n]$ and $w[n]$ are zero-mean and statistically independent. The autocorrelation function of $w[n]$ is
   $E\{w[n]w[n+m]\} = \sigma_w^2\delta[m]$. And the variance of $x[n]$ is σ_x^2.
   (a) Show that $s[n]$ is white.
   (b) Find the mean square of the $s[n]$, $E\{s^2[n]\}$.