Homework 1 (Total 170 pts) Due 5:00 pm on June 11, 2025

Note: You work must be electronically submitted to Canvas as a single PDF file.

- (20 pts) A system has the input-output relation given by y[n] = T{x[n]} = nx[n].
 Determine whether the system is

 (a) linear;
 (b) time invariant;
 (c) stable;
 (d) causal.
- 2. (20 pts) Let $x[n] = \delta[n] + 2\delta[n-1] \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Compute the following convolution: y[n] = x[n] * h[n].

3. (40 pts) Consider the LTI system with impulse response: $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Given an input to the system: $x[n] = (-1)^n u[n]$,

- (a) Determine the steady-state response, i.e., the response of the system to the input when $n \to \infty$.
- (b) Use the *conv* function on Matlab to calculate the output y[n] numerically, where n = 0:100. Stem plot the waveform of the output y[n]. Attach the plot. What is the numerical value of y[55]? Does this value agree with the result you obtained analytically in (a)? Comment on this comparison. Attach your Matlab script used.
- 4. (30 pts)
 - (a) Calculate the Fourier transform of $X(e^{j\omega})$ of the sequence: x[n] = u[n+3] - u[n-4].
 - (b) In Matlab, plot and label the magnitude of the Fourier transform $|X(e^{j\omega})|$ (where ω goes from -2π to 2π . Attach a printed hardcopy of the plot.
- 5. (20 pts)
 - (a) If x[n] is a complex sequence with Fourier transform being $X(e^{j\omega})$, prove that $\mathcal{F}\{x^*[-n]\} = X^*(e^{j\omega})$.
 - (b) Suppose x[n] is a real sequence. Use Property 7 of the Symmetry of Fourier Transforms to prove that the imaginary part of its Fourier transform is odd, i.e., $X_I(e^{j\omega}) = -X_I(e^{-j\omega})$.
- 6. (10 pts) Determine the frequency response $H(e^{j\omega})$ of the LTI system whose input and output satisfy the following difference equation.

$$y[n] + \frac{1}{2}y[n-1] + \frac{3}{4}y[n-2] = x[n] - \frac{1}{2}x[n-1] + x[n-3].$$

7. (10 pts) Suppose the frequency response of an LTI system is given by

$$H(e^{j\omega}) = \frac{1}{1 - 0.5e^{j\omega}}$$
. If the input to the system is $x[n] = 1 + e^{j\pi n}$, what is the output $y[n]$?

- 8. (20 pts) The sequences s[n], x[n] and w[n] are sample sequences of wide-sense stationary random processes where s[n] = x[n]w[n]. The sequences x[n] and w[n] are zero-mean and statistically independent. The autocorrelation function of w[n] is E{w[n]w[n + m]} = σ_w²δ[m]. And the variance of x[n] is σ_x².
 (a) Show that s[n] is white.
 - (b) Find the mean square of the s[n], $E\{s^2[n]\}$.