

Homework 2

(Total 100 pts)

Due 5:00 pm on June 20, 2024 (Thursday)

Note: Your work must be electronically submitted to Canvas as a single **PDF** file.

1. (10 pts) Determine the z -transform, including the ROC for the following sequence:

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n - 1].$$

2. (20 pts) Determine the z -transform of the following sequence. Include with your answer the region of convergence in the z -plane and a sketch of the pole-zero plot. Express all sums in closed form; α can be complex.

$$x_\alpha[n] = \alpha^{|n|}, \text{ where } 0 < |\alpha| < 1.$$

3. (60 pts) A causal LTI system has the following system function:

$$H(z) = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})}$$

- (A) What is the ROC for $H(z)$?
- (B) Is the system stable or not?
- (C) Determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$.
- (D) Determine the impulse response of the system $h[n]$.
- (E) Find $Y(z)$, the z -transform of the output, when the input $x[n] = u[-n - 1]$. Specify the ROC for $Y(z)$.
- (F) Find the output sequence $y[n]$ when the input is $x[n] = u[-n - 1]$.

4. (10 pts) A causal LTI system with input $x[n]$ and output $y[n]$ is described by $y[n] = 2y[n - 1] + x[n]$. Find the impulse response of the system $h[n]$. Is the system stable?