Homework 3

(Total 170 pts)

Due 5:00 pm on July 17, 2024 (Wednesday)

Note: Submit two files ('hw3.pdf' and 'q6.m') on Canvas.

1. (20 pts) The continuous-time signal $x_c(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal

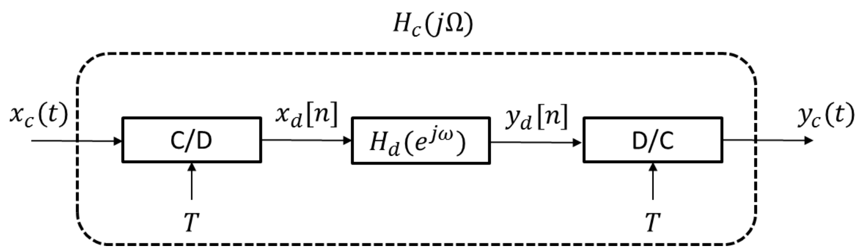
$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right).$$

Determine if the following sampling periods will allow us to convert $x_c(t)$ into $x[n]$:

(a) $T = 0.01$

(b) $T = 0.11$

2. (30 pts)
Consider the system shown below for processing of the input bandlimited continuous-time waveform $x_c(t)$.



Find the continuous-time frequency response $H_c(j\Omega)$, given that

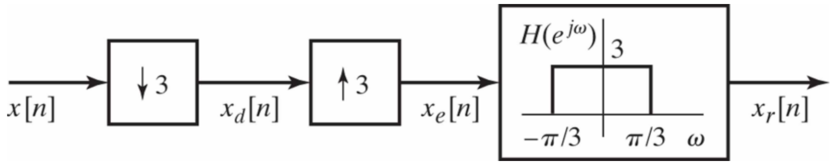
(a) $x_c(t)$ is bandlimited to $|\Omega| < \Omega_M$.

(b) The C/D converter has sampling rate $T = \frac{\pi}{\Omega_M}$, and produces the signal $x_d[n] = x_c(nT)$.

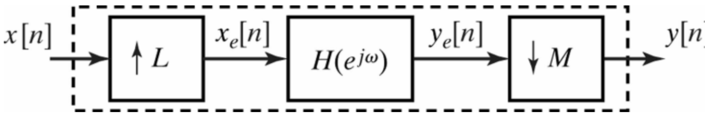
(c) The discrete-time filter has frequency response $H_d(e^{j\omega}) = \frac{e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}}}{T}, \quad |\omega| \leq \pi.$

(d) The ideal D/C converter is such that $y_d[n] = y_c(nT)$.

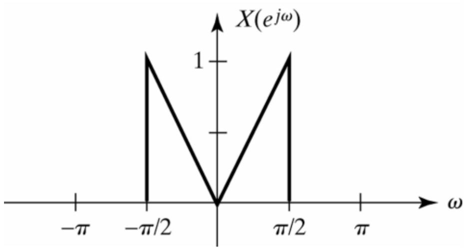
3. (20 pts) Consider the system shown below. For the input signals $x[n] = \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}$. Determine whether the output $x_r[n] = x[n]$. Justify your answer.



4. (40 pts) Consider the discrete-time system shown below



Assume that $L = 2$ and $M = 4$, and that $X(e^{j\omega})$, the DTFT of $x[n]$, is real and is as shown below.

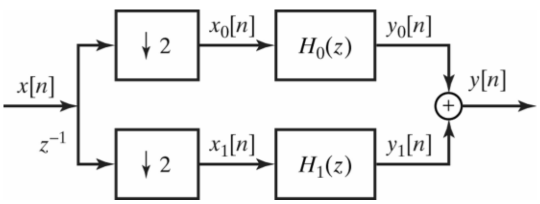


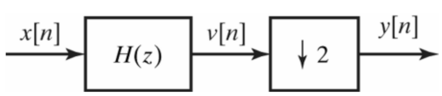
And

$$H(e^{j\omega}) = \begin{cases} M & |\omega| \le \frac{\pi}{4}, \\ 0 & \frac{\pi}{4} < |\omega| \le \pi. \end{cases}$$

Sketch $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$, $Y(e^{j\omega})$, the DTFTs of $x_e[n]$, $y_e[n]$, $y[n]$, respectively. Be sure to clearly label salient amplitudes and frequencies.

5. (20 pts) Consider the decimation filter bank shown below in the left. The filter bank can also be implemented as shown below in the right.





Assume that $y_0[n]$ and $y_1[n]$ are generated according to the following difference equations:

$$y_0[n] = \frac{1}{4} y_0[n-1] - \frac{1}{3} x_0[n] + \frac{1}{8} x_0[n-1]$$

$$y_1[n] = \frac{1}{4} y_1[n-1] + \frac{1}{12} x_1[n]$$

and $v[n] = av[n-1] + bx[n] + cx[n-1]$. Determine a, b, and c.

6. (40 pts) Given a causal LTI system with the system function: $H(z) = \frac{1-2z^{-1}}{1-0.75z^{-1}}$,
   (a) Determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$.
   (b) If the input to this system is $x[n] = 5^n u[n] + 20^n u[-n-1]$, write a Matlab script 'q6.m' to recursively calculate the numerical values of $y[n]$, based on the difference equation obtained in (a), where n goes from 0 to 30, and $y[0] = 0$.
   (c) Attach the source code 'q6.m' in the PDF file 'hw3.pdf'. Also submit 'q6.m' separately on Canvas.
   (d) Show and attach the stem plot of $y[n]$, where n goes from 0 to 30.
   (e) What is the numerical value of $y[30]$?