Homework 4

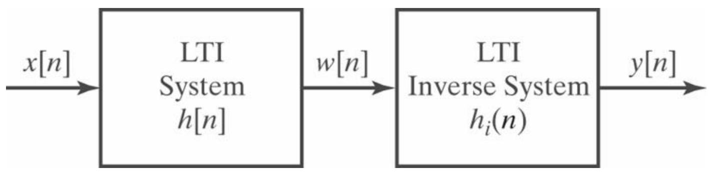
(Total 240 pts)

Due 5:00 pm on July 26, 2024 (Friday)

Note: Submit two files ('hw4.pdf' and 'q5.m') on Canvas.

1. (20 pts) The impulse response of an LTI system is $h[n] = \delta[n] + \delta[n-4]$.
   (a) Determine analytically the group delay associated with the system. Show your derivations.
   (b) Use the *grpdelay* function in Matlab to verify your answer in (a). Attach the Matlab scripts and the plot for group delay.

2. (20 pts) Consider the cascade of an LTI system with its inverse system shown below:



   The impulse response of the first system is $h[n] = \delta[n] + 2\delta[n-1]$.
   (a) Determine the impulse response $h_i[n]$ of a stable inverse system for $h[n]$.
   (b) Is the inverse system causal?

3. (60 pts) A causal LTI system has the system function

$$H(z) = \frac{(1-e^{\frac{j\pi}{3}}z^{-1})(1-e^{-\frac{j\pi}{3}}z^{-1})(1+1.1765z^{-1})}{(1-0.9e^{\frac{j\pi}{3}}z^{-1})(1-0.9e^{-\frac{j\pi}{3}}z^{-1})(1+0.85z^{-1})}.$$

   (a) Sketch the pole-zero diagram. You can use the *zplane* function in Matlab. But make sure you mark the values of the poles and zeros on the plot.
   (b) What is the ROC for the system function?
   (c) Plot the magnitude and phase response of the system using the *freqz* function in Matlab.
   (d) Check whether the following statements are true or false about the system. Justify your answers.
      (i) The system is stable. **True ( ) False ( ). Why?**
      (ii) The system is a minimum-phase system. **True ( ) False ( ). Why?**

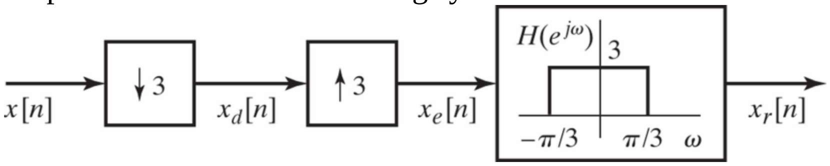
4. (40 pts) Consider the stable LTI system with system function

$$H(z) = \frac{1-4z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}}.$$

   The system function $H(z)$ can be factored such that $H(z) = H_{min}(z)H_{ap}(z)$, where $H_{min}(z)$ is a minimum-phase system, and $H_{ap}(z)$ is an all-pass system, where $|H_{ap}(e^{j\omega}) = 1|$.
   (a) Sketch the pole-zero diagram of $H(z)$.
   (b) Determine $H_{min}(z)$ and its ROC.

(c) Determine $H_{ap}(z)$ and its ROC.
(d) Sketch the pole-zero diagrams of $H_{min}(z)$ and $H_{ap}(z)$.

5. (100 pts) Matlab implementation of the following system we considered in HW3:



(a) Generate a discrete-time input sequence $x[n] = \frac{1}{4} sinc\left(\frac{n}{4}\right)$, for $n = -1000$ to 1000, with a step size of 1. Visualize this sequence $x[n]$ by using the *plot* command. Show and attach the plot.
(b) Generate the sequence $x_e[n]$ by down-sampling and then up-sampling the input sequence, as shown in the system above. Show the plot of $x_e[n]$. Attach the plot.
(c) Implement the filter with $H(e^{j\omega})$ as shown in the above system, by using the *fir1* function in Matlab. See the page below for more details about this function: https://www.mathworks.com/help/signal/ref/fir1.html.
Use a filter order of *nfilt* $= 50$. Show the plot of the resulting vector of filter coefficients obtained. Attach the plot.
(d) Show the frequency response of the above filter using the *freqz* command. Attach the plot.
(e) Obtain the reconstructed sequence $x_r[n]$ by filtering the sequence $x_e[n]$ using the filter coefficients obtained in (c). Use the *filter* function in Matlab for this sake. See the page below for more details about this function:
https://www.mathworks.com/help/matlab/ref/filter.html
(f) Show in the same plot the input and reconstructed sequences using:
figure; plot(x); hold on; plot(xr, 'r')
Attach the plot.
(g) Determine the delay introduced by the filter implemented in (c) by using the *grpdelay* function in Matlab. Let *avg_delay* denote the average group delay over all frequencies considered. What is the value of *avg_delay*?
Then compensate the delay introduced by the filter using:
x_trunc = x(1: end – avg_delay);
xr_shift = xr((avg_delay + 1): end);
Show in the same plot the truncated input sequence (*x_trunc*) and the reconstructed sequence that is delay-compensated (*xr_shift*). Attach the plot.
(h) Calculate the Mean Square Error (MSE) between the truncated input sequence (*x_trunc*) and the delay-compensated reconstructed sequence (*xr_shift*).
As an example of MSE calculation, the MSE between the following two 3-point sequences: [1 2 3], and [4 6 8] is $\frac{50}{3} \approx 16.67$.
(i) Now, experiment with various filter orders such that *nfilt* $= 10, 20, 50, 100, 200$. Repeat steps (c) through (h) to obtain the corresponding MSE values between the truncated input sequence (*x_trunc*) and the delay-compensated reconstructed sequence (*xr_shift*). Fill in the table below with your answers.

Filter Order	10	20	50	100	200
MSE					

(j) What observations can you make regarding the relations between the filter order and the input reconstruction error, as well as the average group delays introduced by filtering?
(k) Attach your MATLAB scripts used and also submit them in a single file (‘q5.m’) to Canvas.