Lecture 1

Discrete-Time Signals and Systems

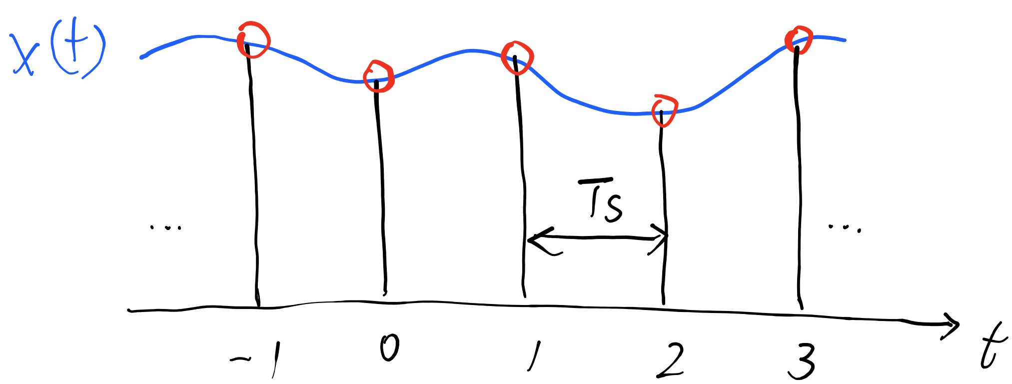
Signals: Functions of variables (Time, Time Index)

$x(t)$ $\downarrow$ Continuous - time

$x[n]$ $\downarrow$ discrete - time, where n is an integer

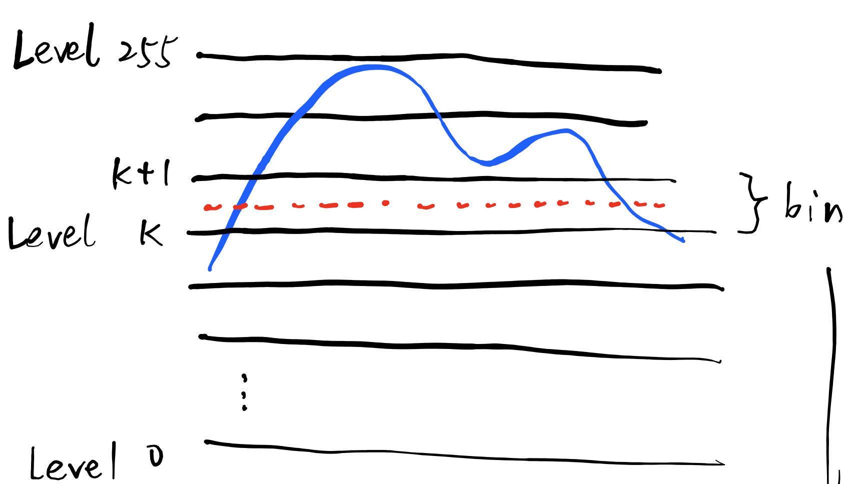
Digital Signals: discrete both in time and amplitude

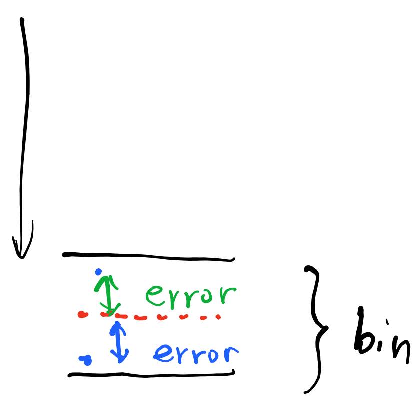
Digitization : $\begin{cases} \text{Sampling} : & x(t) \rightarrow x[n] \\ \text{Quantization} : & x[n] \rightarrow \text{digital signals} \end{cases}$



Ts : Sampling Period , $f_s = \frac{1}{T_s}$

Quantization (e.g., digital images , pixel intensity $\underbrace{0 \sim 255}_{\text{8 bits binary representation}}$)





Quantization : real number $\rightarrow$ k

Dequantization (reconstruction) k $\rightarrow$ midpoint of the bin

Systems

Continuous - Time :

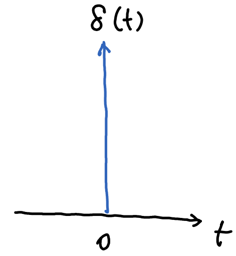
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = \underset{\text{operator on the input}}{T}\{x(t)\}$$

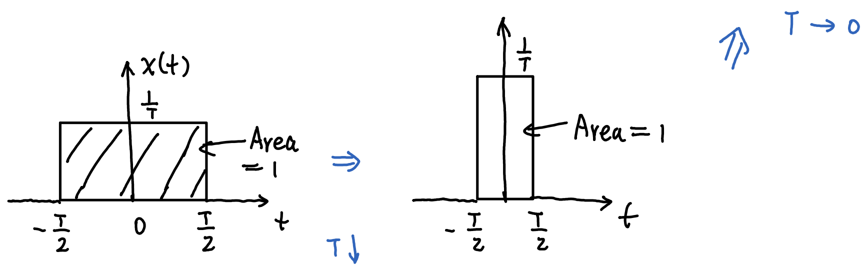
$h(t)$ → Impulse Response

$\underbrace{\text{Impulse}}_{?}$

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$





$$h(t) = T\{x(t) = \delta(t)\}$$

Property of impulse signal $\delta(t)$:

$$\int_{-\infty}^{\infty} \delta(t) \cdot \underbrace{s(t)}_{\text{testing function}}\, dt = s(0)$$

Discrete - Time Systems

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] ,$$

$$h[n] = T\{x[n] = \delta[n]\} , \qquad \delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

... 0, 0, 0, 1, 0, 0, 0, ...

$h[n]$: Impulse Response Sequence

$\delta[n]$: unit impulse sequence

- Linear Time Invariant (LTI) Systems

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$*$: Convolution Sum

$$= h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$

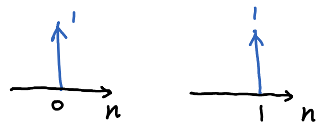
- Unit Step Sequence

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\begin{array}{lccccccccccc} u[n]: & \cdots 0, & 0, & 0, & 1, & 1, & 1, & 1, & \cdots \\ & & & & \uparrow & & & & \\ n & \cdots & -2 & -1 & 0 & 1 & 2 & 3 & \cdots \end{array}$$

Represent $u[n]$ by using $\delta[n]$,

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots$$



In general, any sequence : $$x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k]$$

$$= x[0] \cdot \delta[n] + x[1] \cdot \delta[n-1]$$
$$+ x[2] \cdot \delta[n-2] + \cdots$$
$$+ x[-1] \cdot \delta[n+1] + x[-2] \cdot \delta[n+2]$$
$$+ \cdots$$

Go back to LTI system:

$$y[n] = T\{x[n]\} = T\left\{ \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k] \right\}$$

Linear : $$= \sum_{k=-\infty}^{\infty} x[k] \cdot T\{\delta[n-k]\}$$

TI : $$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] = x[n] * h[n]$$

Why? $$T\{\delta[n]\} = h[n]$$
$$T\{\delta[n-k]\} = h[n-k]$$

Exponential Sequences

$$x[n] = |A| \cdot e^{j(\omega_0 n + \phi)} \quad , \quad \text{where } A: \text{ complex number}$$

$$= |A| \cos(\omega_0 n + \phi) + j|A| \sin(\omega_0 n + \phi)$$

e.g., $x[n] = \underset{\substack{\uparrow \\ \text{Real number}}}{A} e^{j\omega_0 n} = A\cos(\omega_0 n) + jA\sin(\omega_0 n)$

If $\omega_0 = 1$, $y[n] = \sin(n)$

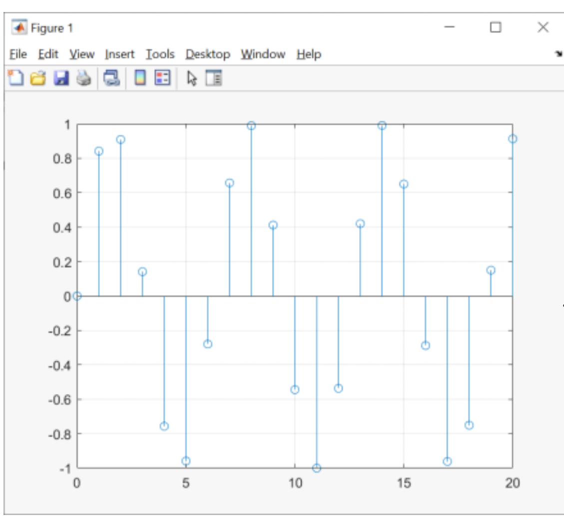
Periodicity: If $y[n]$ is periodic, then $y[n] = y[n+N]$, for all n.

N: period. (integer)

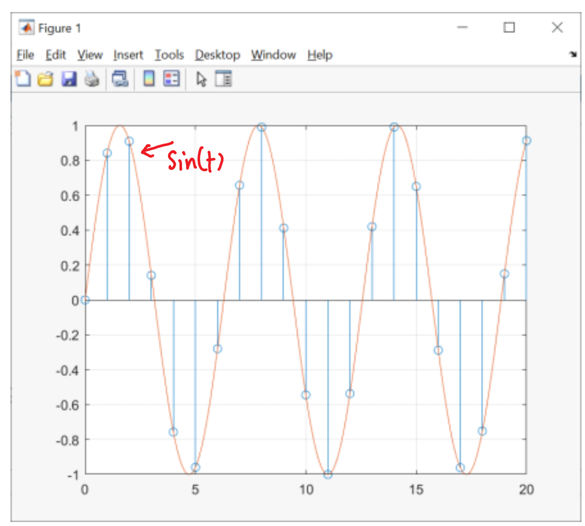
We know that $\sin(t)$ is periodic, since $\sin(t) = \sin(t + 2\pi)$

How about $\sin[n]$, is it periodic?

sin[n] is not periodic



sin(t) is periodic

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>> n = 0: 20;
>> y = sin(n);
>> stem(n, y); grid
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>> t = 0: 0.01: 20;
>> plot(t, sin(t))
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Another sequence: $x[n] = \cos\left(\frac{\pi}{4} n\right)$

$$= \cos\left(\frac{\pi}{4} n + 2\pi\right) = \cos\left[\frac{\pi}{4}(n + \underset{\substack{\uparrow \\ N=8}}{8})\right] \text{, periodic.}$$