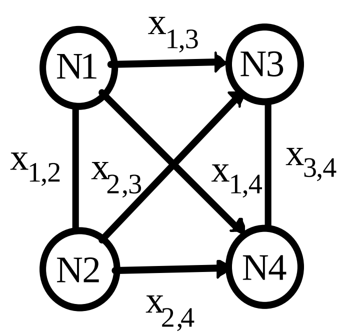
The University of Alabama in Huntsville
Electrical and Computer Engineering
CPE 633 01
Test 2 Solution

1. (20 points) The four node network shown has six links, out of which four are unidirectional and two are bi-directional. Calculate the path reliability for the pair N2-N4.



There are four paths to consider: P1 – $x_{2,4}$, P2 – $x_{1,2}x_{1,4}$, P3 – $x_{2,3}x_{3,4}$, P4 – $x_{1,2}x_{1,3}x_{3,4}$

$$R_{N_s,N_d} = \Pr ob\{E_1\} + \Pr ob\{E_2\}\Pr ob\{\overline{E_1} \mid E_2\} + \ldots + \Pr ob\{E_m\}\Pr ob\{\overline{E_1} \cap \overline{E_2} \cap \cdots \cap \overline{E_{m-1}} \mid E_m\}$$

Prob{P1} = $p_{2,4}$, Prob{P2} = $p_{1,2}p_{1,4}$, Prob{P3} = $p_{2,3}p_{3,4}$, Prob{P4} = $p_{1,2}p_{1,3}p_{3,4}$

$$R_{N_2-N_4} = \Pr ob\{P1\} + \Pr ob\{P2\}\Pr ob\{\overline{P1} \mid P2\} + \Pr ob\{P3\}\Pr ob\{\overline{P1} \cap \overline{P2} \mid P3\}$$
$$+ \Pr ob\{P4\}\Pr ob\{\overline{P1} \cap \overline{P2} \cap \overline{P3} \mid P4\}$$
$$R_{N_2-N_4} = p_{2,4} + p_{1,}2p_{1,4}q_{2,4} + p_{2,3}p_{3,4}q_{2,4}(p_{1,2} + p_{1,4}) + p_{1,2}p_{1,3}p_{3,4}q_{2,4}q_{2,3}q_{1,4}$$

2. (20 points) Derive an approximate expression for the reliability of a square (4, 4) interstitial redundancy array with 16 primary nodes and 9 spares. Denote the reliability of a node by R and assume the links are fault-free.

One approximation is to view it as an 16-out-of-25 structure, i.e.,

$$R_{16\_of\_25} = \sum_{i=0}^{9} (1-R)^i R^{25-i}$$

Another approximation is to claim that four (out of the 16) have one spare, two sets of four nodes have two spares each, and one set of four has four spares. Thus,
$R_{system} = [R^5 + 5R^4(1 - R)][R^6 + 6R^{5(}1 - R) + 15R^4(1 - R)^2]^2[R^8 + 8R^7(1 - R) + 28R^6(1 - R)^2 + 56R^5(1 - R)^3 + 70R^4(1 - R)^4]$.

3. (3 points) Most acceptance tests fall into one of these three categories: __timing checks__, __output verification__, and ___range checks__.

4. (1 point) A well-known multistage network is the _butterfly__.

5. (1 point) Rebooting your PC is an example of __software rejuvenation__.

6. (25 points) Consider the issue of version independence with N-version programming. There are 3 versions of the program and three subspaces to consider. The probability of the input being from subspace S1 is 0.2, from subspace S2 0.5 and from subspace S3 0.3. The conditional failure probabilities are as follows:

Version	S1	S2	S3
V1	0.010	0.009	0.005
V2	0.020	0.004	0.010
V3	0.015	0.016	0.014

(a) What are the unconditional failure probabilities for the three versions?

(b) If the three versions were stochastically independent, what would the probability of them all failing for the same input be?

(c) What is the actual joint failure probability?

(d) Are the versions positively or negatively correlated?

(a) Prob{A} = Σ Prob{A|B_i}·Prob{B_i} over all I

Prob{V1} = Prob{V1|S1}Prob{S1} + Prob{V1|S2}Prob{S2} + Prob{V1|S3}Prob{S3}
= 0.010*0.2 + 0.009*0.5 + 0.005*0.3 = 0.008

Prob{V2} = Prob{V2|S1}Prob{S1} + Prob{V2|S2}Prob{S2} + Prob{V2|S3}Prob{S3}
= 0.020*0.2 + 0.004*0.5 + 0.010*0.3 = 0.009

Prob{V3} = Prob{V3|S1}Prob{S1} + Prob{V3|S2}Prob{S2} + Prob{V3|S3}Prob{S3}
= 0.015*0.2 + 0.016*0.5 + 0.014*0.3 = 0.0152

(b) Prob{V1 ∩ V2 ∩ V3} = Prob{V1}Prob{V2}Prob{V3} = 0.008*0.009*0.0152 = 1.0944 E-6

(c) Prob{A ∩ C} = ΣProb{A|B_i}·Prob{C|B_i}·Prob{B_i} over all i

Prob{V1 ∩ V2 ∩ V3} = Prob{V1|S1}Prob{V2|S1}Prob{V3|S1}Prob{S1}
+ Prob{V1|S2}Prob{V2|S2}Prob{V3|S2}Prob{S2}
+Prob{V1|S3}Prob{V2|S3}Prob{V3|S3}Prob{S3}
= 0.010*0.020*0.015*0.2 + 0.009*0.004*0.016*0.5 + 0.005*0.010*0.014*0.3
= 1.098 E-6

(d) Since the number found in c is greater than the one found in b, the versions are positively correlated.

7. (10 points) Use checksums to detect and correct errors in a scalar by matrix multiplication for the 4 x 4 matrix shown.

$$A = \begin{bmatrix} 1 & 11 & 17 & 6 & 19 \\ 15 & 2 & 12 & 18 & 7 \\ 8 & 16 & 3 & 13 & 20 \\ 10 & 9 & 5 & 4 & 14 \end{bmatrix}$$

Use the corresponding column-weighted matrix AC and assume that during the multiplication of AC by the scalar 4 a single error has occurred resulting in the following output:

$$4 \bullet A = \begin{bmatrix} 4 & 44 & 68 & 24 & 76 \\ 60 & 8 & 48 & 72 & 28 \\ 32 & 64 & 12 & 52 & 80 \\ 39 & 36 & 20 & 16 & 56 \end{bmatrix}$$

The column weighted matrix is

$$A_C = \begin{bmatrix} 1 & 11 & 17 & 6 & 19 \\ 15 & 2 & 12 & 18 & 7 \\ 8 & 16 & 3 & 13 & 20 \\ 10 & 9 & 5 & 4 & 14 \\ 34 & 38 & 37 & 41 & 60 \\ 143 & 151 & 93 & 126 & 225 \end{bmatrix}$$

$$4A_C = \begin{bmatrix} 4 & 44 & 68 & 24 & 76 \\ 60 & 8 & 48 & 72 & 28 \\ 32 & 64 & 12 & 52 & 80 \\ 39 & 36 & 20 & 16 & 56 \\ 136 & 152 & 148 & 164 & 240 \\ 572 & 604 & 372 & 504 & 900 \end{bmatrix}$$

S1

Column 1: 136-4-60-32-39 = 1, Column 2: 152-44-8-64-36 = 0, Column 3: 148-68-48-12-20 = 0, Column 4: 164-24-72-52-16 = 0, Column 5: 240-76-28-80-56 = 0

For columns 2,3,4 and 5 , S_1 and S_2 are both zero. For column 1 we calculate $S_1 = \Sigma^3_{i=1} a_{i,2} - WCS_1$ = (4 + 60 + 32 + 39) -136 = -1, and $S_2 = \Sigma^3_{i=1} 2^{i-1}a_{i,2} - WCS_2$ = (4 + 120 + 128 + 312) - 572 = -8. Since both S_1 and S_2 are non-zero, we calculate $S_2/S_1 = -8/-1 = 2^{(4-1)}$ implying that $a_{4,1}$ is erroneous. We correct the error using $a_{3,2} = a_{3,2} - S_1$ = 39 – (-1) = 40.

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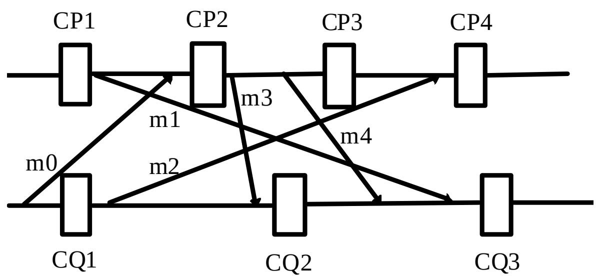
8. (20 points) Identify all the consistent recovery lines in the following execution of two concurrent processes.



Possible recovery lines include:

CP1-CQ1	consistent
CP1-CQ2	m3 orphaned
CP1-CQ3	m1, m3, m4 orphaned

CP2-CQ1	consistent
CP2-CQ2	m3 orphaned
CP2-CQ3	m3, m4 orphaned
CP3-CQ1	consistent
CP3-CQ2	consistent
CP3-CQ3	consistent
CP4-CQ1	m2 orphaned
CP4-CQ2	consistent
CP4-CQ3	consistent